UNCOMPLEMENTED FUNCTION ALGEBRAS WITH SEPARABLE ANNIHILATORS

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Let S be a compact space, A a closed subalgebra of the algebra $C(S)$ of all complex valued continuous functions on S. (By compact we will mean compact and Hausdorff.) We shall say that A is complemented in $C(S)$ if there exists a projection (i.e. bounded linear idempotent operator) from $C(S)$ onto A. Glicksberg [8] conjectured that if A is a complemented subalgebra of $C(S)$, then A is a self adjoint, i.e. if $f \in A$, then $\bar{f} \in A$ where $\bar{f}(s) = \overline{f(s)}$ for $s \in S$. He showed [8; 129] that this conjecture is true in the case where S is a compact topological group and A is an arbitrary translation invariant subalgebra of $C(S)$. This generalized a result due to Rudin [16] (see also [6; 155]) that if K is the unit circle $|z| = 1$, then the "disc algebra" of functions on K having continuous extensions to $|z| \leq 1$ analytic for $|z| < 1$ is uncomplemented in $C(K)$. (Slightly stronger results are obtained in [12, Proposition 2] and in [15].) A "dual" result for Hardy class H_1 is due to Newman [11].

In the present paper we establish Glicksberg's conjecture for subalgebras of $C(S)$ with separable annihilator. If X is a closed subspace of a Banach space Y and Y^* denotes the dual space to Y, then subspace

$$X^{\perp} = \{y^* \in Y^* : y^*x = 0 \text{ for } x \in X\}$$

of the space Y^* is called the annihilator of X. The main result of the present paper is the following

Theorem. *Let A be a closed complemented subalgebra of $C(S)$, S compact. Let $A^{\perp}$ be a norm-separable subspace of $[C(S)]^* = M(S)$. Then A is self-adjoint.*

Clearly, by the F. and M. Riesz theorem [6; 47] the annihilator of the disc algebra is separable. Therefore our theorem implies the result of Rudin mentioned above.

We recall that space $M(S)$, the dual to the space $C(S)$ consists of all regular Borel measures on S. For μ in $M(S)$ we shall employ the notation $\mu(f) = \int f d\mu$.

The proof of the theorem is based upon some results due to Grothendieck [9] concerning Banach spaces with the Dunford-Pettis property (see §1). These allow us (§3) to reduce the problem to the case where the annihilator is purely atomic, i.e. every measure in the annihilator is purely atomic. This case is considered in §2. It is shown that under a certain additional assumption (probably superfluous!) every closed subalgebra with purely atomic annihilator is self-adjoint.

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