q-IDENTITIES OF AULUCK, CARLITZ, AND ROGERS

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1. Introduction. In [1] and [2], a large number of q-identities have been deduced from a single identity of basic hypergeometric type. In particular, most of the third order mock theta function identities [14; 63], all of the fifth order mock theta function identities [15; 277–279], several of the identities given by Fine in [7], and Heine's original transformation of basic hypergeometric series [8; 106] are all deducible from the Fundamental Lemma of [1]. The object of this paper is to show that many other q-identities which have not seemed to fit into any of the general theorems on basic hypergeometric series (c.f. Sears [11], [12], or Slater [13]) are actually deducible from the Fundamental Lemma of [1]. We shall utilize the following notation

$$\prod_m (x, q) = \prod_{j=0}^{m-1} (1 + xq^j)$$

$$\prod_\infty (x, q) = \prod_{j=0}^{\infty} (1 + xq^j).$$

We shall prove

$$\text{(A1)} \qquad \sum_{m=0}^{\infty} \frac{q^{(m+1)^2}}{\prod_m (-q, q) \prod_{m+1} (-q, q)} = q[\prod_\infty (-q, q)]^{-1} \sum_{p=0}^{\infty} (-1)^p q^{\frac{1}{2}p(p+3)},$$

$$\text{(A2)} \qquad \sum_{m=0}^{\infty} \frac{q^{m+1}}{\prod_m (-q, q) \prod_{m+1} (-q, q)} = q[\prod_\infty (-q, q)]^{-2} \sum_{p=0}^{\infty} (-1)^p q^{\frac{1}{2}p(p+3)},$$

$$\text{(A3)} \qquad \sum_{m=0}^{\infty} \frac{q^{\frac{1}{2}(m+1)(m+2)}}{\prod_m (-q, q) \prod_{m+1} (-q, q)} = q[\prod_\infty (-q, q)]^{-1} \sum_{m=0}^{\infty} \frac{q^{2m^2+3m}}{\prod_m (-q^2, q^2)}$$

$$\text{(C1)} \qquad \sum_{s=0}^{\infty} \frac{\prod_s (-b, q) x^s}{\prod_s (-q, q) \prod_s (-a, q)} = \frac{\prod_\infty (-bx, q)}{\prod_\infty (-a, q) \prod_\infty (-x, q)} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{1}{2}n(n-1)} a^n \prod_n (-x, q)}{\prod_n (-q, q) \prod_n (-bx, q)}$$

$$\text{(C2)} \qquad \sum_{n=0}^{\infty} \frac{q^{\frac{1}{2}n(n+1)} \prod_n (-a, q)}{\prod_n (-q, q)} = \prod_\infty (q, q) \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2} a^n}{\prod_n (-q^2, q^2)}$$

$$\text{(C3)} \qquad \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2} \prod_n (-a, q^2)}{\prod_n (-q^2, q^2)} = \prod_\infty (-q, q^2) \sum_{n=0}^{\infty} \frac{q^{2n^2-n} a^n}{\prod_{2n} (-q, q)}$$

$$\text{(R1)} \qquad \sum_{n=0}^{\infty} \frac{q^{4n^2} z^{2n}}{\prod_n (-q^4, q^4)} = \prod_\infty (-zq, q^2) \sum_{n=0}^{\infty} \frac{q^{n^2} z^n}{\prod_n (-q^2, q^2) \prod_n (-zq, q^2)}$$

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