A GENERAL CLASS OF ARITHMETICAL FUNCTIONS

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Let $g(n)$ be an arbitrary arithmetical function for which $g(0) = 0$ and $g(n) \geq g(n-1)$ for $n \geq 1$ and define

$$a(n) = \sum_{\nu=1}^{r} g(\alpha_\nu)$$

for $n > 1$ and $a(1) = 0$, where $n = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$ is the representation of n as a product of powers of distinct primes. Thus $g(n) \geq 0$ for $n \geq 0$ and $a(n) \geq 0$ for $n \geq 1$. The function $a(n)$ is a generalization of some functions considered previously [1], [2], [3], [4].

To derive the following results about $a(n)$, we impose the final condition

$$g(n) = o(n^{-\lambda}2^n)$$

concerning the order of magnitude of $g(n)$. This condition, as well as the previous conditions, will be used many times in the sequel without explicit mention.

THEOREM 1:

$$\sum_{n \leq x} a(n) = g(1)x \log\log x + Bx + o\left(\frac{x}{\log^{\lambda-1} x}\right) \text{ for } 1 < \lambda < 2;$$

$$\sum_{n \leq x} a(n) = g(1)x \log\log x + Bx + o\left(\frac{x}{\log x}\right) \text{ for } \lambda \geq 2,$$

where

$$B = g(1)\gamma + \sum_{p} \{(1 - p^{-1})G(p^{-1}) + g(1)\log(1 - p^{-1})\};$$

$$G(x) = \sum_{n=1}^{\infty} g(n)x^n \text{ and } \gamma \text{ is Euler's constant.}$$

PROOF.

$$\sum_{n \leq x} a(n) = \sum_{n \leq x} \sum_{\nu=1}^{r} g(\alpha_\nu) = \sum_{n \leq x} \sum_{p^m | n} \{g(m) - g(m-1)\}$$

$$= \sum_{p^m \leq x} \{g(m) - g(m-1)\}\left[\frac{x}{p^m}\right]$$

$$= g(1) \sum_{p \leq x} \left[\frac{x}{p}\right] + \sum_{\substack{p^m \leq x \\ m \geq 2}} \{g(m) - g(m-1)\}\left[\frac{x}{p^m}\right]$$

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