A CHARACTERIZATION OF THE SET OF ASYMPTOTIC VALUES OF A FUNCTION HOLOMORPHIC IN THE UNIT DISC

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1. The set of asymptotic values, or *asymptotic set*, of a function meromorphic in $\mathcal{U} = \{|z| < 1\}$ is characterized as an analytic subset of the extended complex plane $\mathcal{W}$ [1], [5]. However, easy examples show that there exist analytic sets in the plane which cannot be the asymptotic set of any function *holomorphic* in $\mathcal{U}$, for example the analytic set $\{0 \leq x \leq 1\}$. In the previous note a characterization of the asymptotic set of a holomorphic function mapping $\mathcal{U}$ onto itself was obtained:

THEOREM 1. *A set $\mathcal{A}$ is the asymptotic set of a holomorphic function f mapping $\mathcal{U}$ onto itself if, and only if, $\mathcal{A}$ is an analytic subset of $\mathcal{U}^-$ and for every r, $0 < r < 1$, there exists a holomorphic function f_r mapping $\mathcal{U}$ into itself with the properties*

(a) *$\mathcal{A}$ contains the asymptotic set of f_r,*
(b) *f_r maps a Jordan region topologically onto $\{|w| \leq r\}$.*

Our present objective is to extend this characterization to unrestricted functions holomorphic in $\mathcal{U}$. We shall find that this characterization also serves for functions which are normal holomorphic in the sense of Lehto and Virtanen [2], and hence for functions in MacLane's class $\mathcal{A}$ [4].

The following two sections introduce notation and facts used to establish the main result, Theorem 4. In addition, Theorem 3 extends results of MacLane [4] and McMillan [6] concerning the class $\mathcal{A}$. We conclude with a result concerning normal holomorphic functions.

2. If S is a set then S^-, S', and ∂S denote its closure, complement, and boundary. If f is a complex valued function defined in some domain, then $\mathcal{A}(f)$ denotes the asymptotic set of f. In the previous note we obtained some necessary conditions on $\mathcal{A}(f)$ for f holomorphic in $\mathcal{U}$, namely,

THEOREM 2. *If f is holomorphic in $\mathcal{U}$ with $f(\mathcal{U}) = D$, then*

(a) *$\mathcal{A}(f)$ is analytic,*
(b) *$\partial D \subset \mathcal{A}(f)^- \subset D^-$,*
(c) *if $\zeta \in \partial D$ is inaccessible from D, then $\zeta \notin \mathcal{A}(f)$,*
(d) *if $\zeta \in \partial D - \mathcal{A}(f)$ is accessible from D, then every arc in D to ζ must meet $\mathcal{A}(f)$.*

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