AREA AND MOMENT OF INERTIA THEOREMS FOR CIRCULAR DOMAINS IN C^n

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Introduction. Let D be a circular domain in the space C^n of n complex variables $z = (z_1, \cdots, z_n)$ with center at the origin, that is, if $z^0 = (z_1^0, \cdots, z_n^0) \in D$, then $e^{i\theta}z^0 \in D$ for arbitrary real θ and $0 = (0, \cdots, 0) \in D$. A circular domain is *complete* if $z \in D$ implies that $rz \in D$ for $0 \leq r \leq 1$. A complete circular domain is star-shaped and univalent.

Let

$$w = f(z) \tag{1}$$

be a holomorphic mapping of D onto a domain $D^* \subset C_n$, that is, $w = (w_1, \cdots, w_n)$, where $w_i = f_i(z_1, \cdots, z_n) \equiv f_i(z)$ are holomorphic functions of z on D. Let

$$J(z) = \partial(f)/\partial(z) \tag{2}$$

be the Jacobian of the mapping (1).

In §1 we generalize a Bieberbach area theorem for the unit disk to a circular domain D. In §2 the set of transformations of the class $\mathfrak{F}$ of volume-preserving, center-invariant, holomorphic mappings which minimize the moment of inertia of a complete circular domain D of finite volume is determined (Theorem 3). Theorem 4 gives a necessary and sufficient condition that such a domain D have a minimal moment of inertia. Myrberg [11] has already considered such area and moment of inertia theorems for the hypersphere in C^2, and we follow his procedure wherever possible. In §3 it is shown that the Cartan domains, that is, the bounded symmetric irreducible domains in C^n satisfy the hypotheses of Theorem 4. Properties of complete orthonormal systems are used in the proofs.

1. Analog of Bieberbach area theorem for circular domains.

1. THEOREM 1. *The holomorphic mapping* (0.1), *normalized by the condition* $|J(0)| = 1$, *maps a circular domain D onto a domain whose volume is in general larger than that of D. For circular domains with finite volume the volume of the image domain is equal to that of D if and only if the absolute value of the Jacobian of the mapping is* 1.

Proof. If the volume of the image domain D^* under (0.1) is infinite the first part of the theorem is true. Assume that the volume of D^* is finite. Its

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