# NOTE ON SOME CONTINUED FRACTIONS OF THE ROGERS-RAMANUJAN TYPE

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1. Making use of the Rogers–Ramanujan identities, it can be shown that [2; 290–295]

$$1 + \frac{x}{1+} \frac{x^2}{1+} \frac{x^3}{1+} \cdots = \prod_1^\infty \frac{(1 - x^{5n-3})(1 - x^{5n-2})}{(1 - x^{5n-4})(1 - x^{5n-1})} \qquad (|x| < 1).$$

Gordon [1] has considered a number of continued fractions analogous to the one on the left side of this equation. In particular he has proved the identity

$$(1) \qquad 1 + x + \frac{x^2}{1 + x^3 +} \frac{x^4}{1 + x^5 +} \frac{x^6}{1 + \cdots}$$

$$= \prod_1^\infty \frac{(1 - x^{8n-5})(1 - x^{8n-3})}{(1 - x^{8n-7})(1 - x^{8n-1})} \qquad (|x| < 1).$$

The proof of (1) depends on the formula

$$(2) \qquad F(a, x) = \frac{P(a, x)}{Q(a, x)},$$

where

$$F(a, x) = 1 + ax + \frac{ax^2}{1 + ax^3 +} \frac{ax^4}{1 + ax^5 +} \frac{ax^6}{1 + \cdots},$$

$$P(a, x) = \sum_0^\infty a^n x^{n^2} \frac{(1 + x)(1 + x^3) \cdots (1 + x^{2n-1})}{(1 - x^2)(1 - x^4) \cdots (1 - x^{2n})},$$

$$Q(a, x) = \sum_0^\infty a^n x^{n^2+2n} \frac{(1 + x)(1 + x^3) \cdots (1 + x^{2n-1})}{(1 - x^2)(1 - x^4) \cdots (1 - x^{2n})}$$

which is valid for $|x| < 1$ and all $a$ such that $Q(a, x) \neq 0$. Note that $Q(a, x) = P(ax^2, x)$.

It may be of interest to consider the slightly more general continued fraction

$$(3) \qquad F(a, b, x) = 1 + ax + \frac{abx^2}{1 + ax^3 +} \frac{abx^4}{1 + ax^5 +} \frac{abx^6}{1 + \cdots}.$$

Let $P_n = P_n(a, b, x)$, $Q_n = Q_n(a, b, x)$ denote the numerator and denominator, respectively of the $n$-th convergent of $F(a, b, x)$. Then

$$P_0 = 1 + ax, \qquad P_1 = 1 + ax + abx^2 + ax^3 + a^2x^4,$$

$$Q_0 = 1, \qquad Q_1 = 1 + ax^3$$

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