SOME GENERATING FUNCTIONS

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The generalized hypergeometric series

$$ {}_2F_0(-n, a-1+n; -; -x/b) $$

is the Bessel polynomial of Krall and Frink, for we know that

$$ (1) \qquad \begin{aligned} y_n(x, a, b) &= \sum_{k=0}^{n} \binom{n}{k} \binom{n+k+a-2}{k} k!(x/b)^k \\ &= {}_2F_0(-n, a-1+n; -; -x/b). \end{aligned} $$

The object of this note is to consider some generating functions of the generalized hypergeometric series

$$ {}_2F_0(-n, c+kn; -; x), $$

where k is a positive integer.

First we observe that

$$ \begin{aligned} \sum_{n=0}^{\infty} {}_2F_0(-n, c+kn; -; x) \frac{t^n}{n!} &= \sum_{n=0}^{\infty} \sum_{i=0}^{n} \frac{(-1)^i (c)_{kn+i}}{i!(n-i)!(c)_{kn}} x^i t^n \\ &= \sum_{n=0}^{\infty} \sum_{i=0}^{\infty} (-1)^i \frac{(d_n)_{(k+1)i}}{(d_n)_{ki}} \frac{x^i t^{n+i}}{n!i!}, \end{aligned} $$

where $d_n = c + kn$. Now we know that if k is a positive integer and n a non-negative integer, then

$$ (2) \qquad (\alpha)_{kn} = k^{nk} \left(\frac{\alpha}{k}\right)_n \left(\frac{\alpha+1}{k}\right)_n \cdots \left(\frac{\alpha+k-1}{k}\right)_n. $$

Thus we derive

$$ (3) \qquad \begin{aligned} &\sum_{n=0}^{\infty} {}_2F_0(-n, c+kn; -; x) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} {}_{k+1}F_k \left[\begin{matrix} \frac{d_n}{k+1}, \frac{d_n+1}{k+1}, \cdots \frac{d_n+k}{k+1}; \\ \frac{d_n}{k}, \frac{d_n+1}{k}, \cdots \frac{d_n+k-1}{k}; \end{matrix} \; -\frac{(k+1)^{k+1}}{k^k} xt \right] \frac{t^n}{n!}. \end{aligned} $$

Next we notice the following formula of Bailey [1]:

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