# OBSTRUCTIONS TO STABLE STRUCTURES ON MANIFOLDS

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Suppose $M$ is a topological manifold. The purpose of this paper is to define an abelian group $G$ and a class $\{c\} \in H^1(M; G)$ which is an obstruction to the existence of a stable structure on $M \times R^k$ (see below for the definition of a stable sturcture). If $M$ has a stable structure, then $\{c\} = 0$. If $\{c\} = 0$ then, for large enough $k$, $M \times R^k$ will have a stable structure, i.e. there are no higher obstructions. A corollary to the theory is that, if $H_1(M; Z) = 0$, then $M \times R^k$ has a stable structure. It is known that simply-connected manifolds admit stable structures [2]. Furthermore, it will be shown that if $S_1$ and $S_2$ are two distinct stable structures on $M \times R^k$, then there exists an integer $p$ such that $S_1$ and $S_2$ induce equivalent structures on $M \times R^k \times R^p$. This fact is independent of the obstruction theory.

Let $T(M) \rightarrow M$ be the topological tangent bundle of $M$. The problem of constructing a stable structure on $M$ is shown to be related to the problem of reducing the group of $T(M) \rightarrow M$ from the group of all homeomorphisms to the subgroup of stable ones. $M$ may be embedded in a high-dimensional euclidian space with a topological normal bundle (see [6], [7] and [8]). Then the total space $K$ of this bundle is a locally-finite complex with the same homotopy type as $M$. Let $E \rightarrow K$ be the pull-back of $T(M) \rightarrow M$. It is for the bundle $E \rightarrow K$ that the obstruction is defined. The approach follows the standard lines of obstruction theory, but the pathological nature of the groups involved prevents it from being a corollary to the classical obstruction theory for the reduction of the group of a bundle.

Let $P$ represent the group of (orientation preserving) homeomorphisms of some euclidian space and $Q$ the stable ones. It will be proved that if two $Q$ bundles are equivalent as $P$ bundles, then for some $k$, $E_1 \times R^k \rightarrow K$ and $E_2 \times R^k \rightarrow K$ are equivalent as $Q$ bundles. This proof is independent of the obstruction theory.

**Background and motivation.** A manifold with a piecewise-linear structure also has a stable structure, although the converse is not known. There are classical conjectures with regard to piecewise-linear structures, namely (1) for some $k$, $M \times R^k$ admits a piecewise-linear structure, (2) two piecewise-linear structures on $M \times R^k$ induce equivalent ones on $M \times R^k \times R^p$, for some $p$. (The second of these implies the topological invariance of the rational Pontryagin classes.) The idea of the present paper is to prove the analogues of these conjectures for stable structures.

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