TAMING 2-COMPLEXES IN HIGH-DIMENSIONAL MANIFOLDS

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1. Introduction. Suppose that S is a k-sphere which is topologically imbedded in the n-sphere S^n, with $n \geq 4$. Recent results have drawn attention to the problem of characterizing the set E of points at which S can fail to be locally flat. J. C. Cantrell [3] has shown that E cannot consist of a single point in the case $k = n - 1$, $n \geq 4$, while J. Stallings [11] has obtained the same result in the case where $k \leq n - 3$, $n \geq 5$. It is plausible to conjecture that the set E can contain no isolated points, and must therefore be uncountable if it is non-empty. However, in neither of the above cases do the methods used suffice to prove this, or even that E cannot consist of two distinct points.

Cantrell and Edwards [2] have shown that a simple closed curve in S^n ($n \geq 4$) is tame if it is locally polyhedral except possibly at a countable number of points. In light of results of T. Homma [8] and H. Gluck [5], this implies that a simple closed curve in S^n ($n \geq 4$) is flat if it is locally flat except possibly at a countable number of points [4]. The present paper was motivated by the corresponding question for imbeddings of 2-spheres in S^n.

It is shown that, if K is a compact 2-complex topologically imbedded in a combinatorial manifold M^n of dimension $n \geq 6$, then K is tame if and only if every simplex of K is tame (Theorem 5). This result is used to prove that K is tame in M^n if it is locally tame except possibly at countably many points (Theorem 6). This, of course, implies that a closed 2-manifold imbedded in M^n is tame if it is locally flat except possibly at countably many points (Theorem 3). All of these theorems fail for imbeddings of 2-complexes in Euclidean 3-space E^3, while their status in dimensions 4 and 5 is as yet undecided.

2. Definitions and preliminary lemmas. A *combinatorial n-manifold* (*n-manifold with boundary*) is a connected separable metric space M^n which is triangulated as a simplicial complex in which the link (star) of each vertex is piecewise-linearly homeomorphic to the boundary of an n-simplex (to an n-simplex). By a *polyhedron* in M^n is meant a subcomplex of a rectilinear subdivision of M^n. If K is a compact complex which is topologically imbedded in M^n, then K is said to be *tame* in M^n if there exists a homeomorphism h of M^n onto itself such that $h(K)$ is a polyhedron. K is said to be *locally tame* (*locally polyhedral*) at the point $p \in K$ if there exists a closed neighborhood U of p in M^n such that $U \cap K$ is tame in M^n (is a polyhedron). K is locally tame (or locally polyhedral) *modulo* the subset A if K is locally tame (or locally polyhedral) at each point of $K - A$. If N is a k-manifold topologically im-

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