TRIGONOMETRIC INTERPOLATION

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1. Recently O. Kiš, [4] has found the explicit form of the trigonometric polynomial $R_n(x)$ of order n for which

$$(1) \qquad R_n^{(m)}(x_k) = \alpha_{km}, \ m = 0, 2; x_k = \frac{2k\pi}{n}, \qquad k = 0, 1, \cdots, n-1$$

are prescribed. He has shown that for n even, such polynomials need not exist but that for n odd, the polynomials of interpolation exist and are unique. In conformity with recent usage, we call it the (0, 2) interpolation. For other interesting results for this type of interpolation through power polynomials on different abscissas, one may refer to J. Balázs and P. Turán [1], [2] J. Surányi and P. Turán [7] and Saxena and Sharma [6]. An earlier paper by H. Poritsky [5] deserves mention, for he considers the (0, m) case for power polynomials. But he is interested in obtaining an analogue of Jacobi expansion on a fixed set of abscissas and in the regions of convergence of the corresponding series.

The object of this paper is to obtain (§2) the explicit form of the trigonometric polynomial $R_n(x)$ of order n and to establish their uniqueness in the (0, M) case, that is, when

$$(2) \quad R_n(x_{k,n}) = \alpha_{kn}, \ R_n^{(M)}(x_{k,n}) = \beta_{kn}, \ x_{k,n} = \frac{2k\pi}{n}, \qquad (k = 0, 1, \cdots, n-1)$$

are prescribed, M being a fixed positive integer ≥ 1. (For the sake of simplicity we shall throughout write x_k, α_k, β_k for x_{kn}, α_{kn}, β_{kn} respectively.) When $M = 1$, the polynomials have been dealt with by Dunham Jackson (see Zygmund [9]) and when $M = 2$, the case has been treated by O. Kiš [4]. (Our Theorem 1 (in the special case $M = 1$) differs in slight detail from Theorem 6.12 [9, Vol. II; 26] where the condition $\sum_{k=0}^{n-1} \beta_k = 0$ is required. In our case the condition is not needed since we are considering polynomials of a higher order as given by (7).) It turns out that the situation is different when M is even from that when M is odd (as was pointed out by Kiš in the special case of $M = 2$) in that when M is even, there does not always exist a polynomial of interpolation when the number n of nodes is even, while in the case of M odd, the interpolatory polynomial always exists for n even or odd.

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