SOME INEQUALITIES FOR GENERALIZED AXIALLY SYMMETRIC POTENTIALS WITH ENTIRE AND MEROMORPHIC ASSOCIATES

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I. Introduction. Solutions to the partial differential equation of generalized axially symmetric potential theory [13],

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{k}{y}\frac{\partial\phi}{\partial y} = 0, \qquad k > 0 \qquad \text{GASPT}$$

may be generated by the integral operator [7], [8], [9], [10]

$$\mathfrak{A}_k[f(\sigma)] \equiv \alpha_k \int_{\mathfrak{L}} f(\sigma)\left[\zeta - \frac{1}{\zeta}\right]^{k-1} \frac{d\zeta}{\zeta},$$

$$(1) \qquad u(z) \equiv u(x, y) = \mathfrak{A}_k[f(\sigma)], \qquad \alpha_k = \frac{4}{(4i)^k\Gamma(k/2)^2};$$

$$\mathfrak{L} \equiv \{\zeta \mid \zeta = e^{i\phi}, \ 0 \le \phi \le \pi\}, \quad |z - z^0| < \epsilon, \quad z = x + iy$$

where z^0 is an initial point of definition, and $\epsilon > 0$ is sufficiently small. The operator $\mathfrak{A}_k[f(\sigma)]$ maps analytic functions of the complex variable σ onto solutions of the GASPT equations $u(x, y)$. Because of this, one is able to study properties of solutions to (1) by considering certain classes of analytic functions; for instance, one may frequently transplant theorems about analytic functions into analogous theorems about GASPT functions. This method for studying solutions of partial differential equations is known as *Bergman's Integral Operator Method* [1], [2], [3].

The function $f(\sigma)$, which is mapped onto a GASPT function $u(x, y)$ by $\mathfrak{A}_k[f]$ is known as the $\mathfrak{A}_k$-associate of $u(x, y)$. In general, theorems obtained about solutions of partial differential equations are given in terms of the associate function; however, in the case of theorems about the domain of regularity, or the singularities of solutions, theorems are given independently of the associate functions. The reason for this is, that by introducing an inverse operator $\mathfrak{A}_k^{-1}[u]$ one may discover which singularities of $f(\sigma)$ actually give rise to singularities of $u(x, y)$. (A discussion of this method has been given in [7] [9].)

The purpose of this note is to obtain inequalities concerning entire and

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