NOTE ON A PAPER OF L. BERNSTEIN

By L. Carlitz

In a recent paper [1] on the modified Jacobi–Perron algorithm, Bernstein introduces a function $P_n(y, z)$ which is defined in the following way:

$$P_n(y, z) = \sum L(s_1, s_2, \cdots, s_y; z)\binom{n}{s_1}\binom{n}{s_2}\cdots\binom{n}{s_y}, \tag{1}$$

where the summation is over all integers $s_1, s_2, \cdots, s_n$ such that

$$s_1 + s_2 + \cdots + s_y = z \qquad (1 \leq s_i \leq n) \tag{2}$$

and

$$L(s_1, s_2, \cdots, s_y; z) = \frac{y!}{A!B!\cdots Y!}, \tag{3}$$

where

$$s_1 = s_2 = \cdots = s_a = A,$$
$$s_{a+1} = s_{a+2} = \cdots = s_b = B,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$s_{r+1} = s_{r+2} = \cdots = s_y = Y.$$

The variables n, y, z are all assumed to be positive integers.

It follows readily from the definition that

$$P_n(y, y) = n^y, \tag{4}$$

$$P_n(y, y+1) = y\binom{n}{2}n^{y-1}, \tag{5}$$

$$P_n(y, ny) = 1, \tag{6}$$

$$P_n(y, y+2) = y\binom{n}{3}n^{y-1} + \binom{y}{2}\binom{n}{2}^2 n^{y-2}, \tag{7}$$

$$P_n(1, z) = \binom{n}{z}. \tag{8}$$

Moreover, Bernstein proves the basic formula

$$\sum_{k=0}^{x-1}\binom{n}{x-k}P_n(y, y+k) = P_n(y+1, y+x) \qquad (1 \leq x \leq n). \tag{9}$$

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