MEROMORPHIC CLOSE-TO-CONVEX FUNCTIONS

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Rodzicom moim, Antoniemu i Katarzynie Libera, w 45 rocznice ich małżeństwa.

1. Introduction. Let g and G be regular in the unit disk $E(|z|<1)$ and satisfy the conditions $g(0) = G(0) = 0$, $g'(0) = 1$ and $G'(0) = e^{i\alpha}$, where α is real. If

$$\text{Re}\left\{\frac{zg'(z)}{G(z)}\right\} \geq \lambda \quad \text{and} \quad \text{Re}\left\{\frac{zG'(z)}{G(z)}\right\} \geq \sigma \tag{1.1}$$

for z in E and $0 \leq \lambda, \sigma \leq 1$, then g is close-to-convex of order λ and type σ with respect to G. This definition and some of its consequences are discussed in [1]. Here we will extend the definition to meromorphic close-to-convex functions [2], [4].

$$F(z) = \frac{e^{i\alpha}}{z} + b_0 + b_1 z + \cdots + b_n z^n + \cdots \ (\alpha \text{ real}), \tag{1.2}$$

regular in the annulus $0 < |z| < 1$ (hereafter called A), is starlike of order σ, $0 \leq \sigma \leq 1$, if and only if

$$\text{Re}\left\{\frac{-zF'(z)}{F(z)}\right\} \geq \sigma, \qquad z \in E. \tag{1.3}$$

This class of functions will be denoted by Σ_σ^*. These functions have been the subject of recent investigations by Ch. Pommerenke [5].

Denote by $\Gamma(\lambda, \sigma)$, $0 \leq \lambda, \sigma \leq 1$, the family of functions

$$f(z) = \frac{1}{z} + a_0 + a_1 z + \cdots + a_n z^n + \cdots \tag{1.4}$$

which are regular in A and together with some $F \in \Sigma_\sigma^*$ satisfy the condition

$$\text{Re}\left\{\frac{zf'(z)}{F(z)}\right\} \geq \lambda, \qquad z \in E. \tag{1.5}$$

If $f \in \Gamma(\lambda, \sigma)$ then we say "f is (meromorphically) close-to-convex of order λ and type σ"; and $f \in \Gamma(\lambda, \sigma)$ w.r.t. F is read "f is close-to-convex of order λ and type σ with respect to F".

If $\lambda \geq \lambda_0$ and $\sigma \geq \sigma_0$, then $\Gamma(\lambda, \sigma) \subseteq \Gamma(\lambda_0, \sigma_0)$ and for all admissible λ and σ, $\Gamma(\lambda, \sigma) \subseteq \Gamma(0, 0) \equiv \Gamma$. Evidently $e^{-i\alpha}F \in \Gamma(\sigma, \sigma)$ if $F \in \Sigma_\sigma^*$. $\Gamma(1, 0)$ is a subclass of the set of meromorphic convex functions.

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