BAER SEMIGROUPS

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1. Introduction. The main object of interest in this paper will be a multiplicatively written semigroup with 0 having the property that the right (respectively left) annihilator of each element is a principal right (respectively left) ideal generated by an idempotent. Such a semigroup is called a *Baer semigroup*. Examples are provided by: (i) the multiplicative semigroup of any Baer ring [7; 2–3]; (ii) any pseudo-complemented semi-lattice [6; 506]; (iii) any Baer *-semigroup [5; 899]. Our main result is that the poset of right (left) annihilators of elements of a Baer semigroup forms a lattice, and that in the presence of certain fairly natural conditions on the semigroup, it is possible to abstractly characterize lattices that arise in this manner.

2. Baer semigroups. Let S be a semigroup with O. Given $x \in S$, let $R(x) = \{y \in S : xy = 0\}$, $L(x) = \{y \in S : yx = 0\}$, $RI(x) = \{e \in S : e = e^2, eS = R(x)\}$, and $LI(x) = \{f \in S : f = f^2, Sf = L(x)\}$. For M a non-empty subset of S, let $R(M) = \{y : my = 0 \text{ for all } m \in M\} = \bigcap_{m \in M} R(m)$. The symbols $RI(M)$, $L(M)$ and $LI(M)$ are defined in the obvious fashion. The condition that S be a Baer semigroup can now be stated in the following form:

(A) For each $x \in S$, both $RI(x)$ and $LI(x)$ are non-empty.

We assume until further notice that S is a Baer semigroup.

The collection of principal right (left) ideals of S forms a poset with set theoretic inclusion as the partial ordering. Note that if $xS = yS$, then $L(x) = L(y)$; if $Sx = Sy$, then $R(x) = R(y)$. Thus the mappings $xS \to L(x)$, $Sx \to R(x)$ are well defined, and clearly induce a Galois connection between the poset of principal right ideals of S and the poset of principal left ideals of S. Let $\mathcal{R}$ (respectively $\mathcal{L}$) denote the set of all right (respectively left) annihilators of elements of S. The situation is summarized in the next lemma. Since each assertion follows in a straightforward manner, and is at any rate well known from the theory of Galois connections (see [1; 56], and [10]), the proof will be omitted.

Lemma 1. *Let x, y, e, $f \in S$ with e, f idempotents. Then*:

(i) $xS \leq yS \Rightarrow L(y) \leq L(x)$; $Sx \leq Sy \Rightarrow R(y) \leq R(x)$.
(ii) $xS \leq RL(x)$; $Sx \leq LR(x)$.
(iii) $L(x) = LRL(x)$; $R(x) = RLR(x)$.
(iv) *The elements of $\mathcal{R}$ (respectively $\mathcal{L}$) are principal right (respectively left) ideals generated by idempotents.*

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