

CERTAIN EXPANSIONS OF GENERALISED BASIC HYPERGEOMETRIC FUNCTIONS

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1. Introduction. Various expansions of generalised hypergeometric functions have been obtained from time to time notably by F. H. Jackson [9], [10]; J. L. Burchnall [6], [7]; T. W. Chaundy [6], [7], [8]; L. J. Slater [14]; R. P. Agarwal [3]; F. M. Ragab [11, 12] and N. Agarwal [1, 2]. The object of this paper is to derive four general expansions involving generalised basic hypergeometric series which include, as special cases, many of the expansions obtained by the previous authors. In the sequel, certain expansions, which are believed to be new, are also derived. The paper is concluded with a brief discussion of the convergence conditions of the expansion theorems given in the paper.

2. Notation. The following usual notation is used throughout this paper. Let, for $|q| < 1$

$$[a]_r = [q^a]_r = (1 - q^a)(1 - q^{a+1}) \cdots (1 - q^{a+r-1});$$

$$[a]_0 = [q^a]_0 = 1; \quad [a]_{-r} = \frac{(-1)^r q^{r/2(r+1)}}{q^{ar} [1 - a]_r}.$$

Then the generalised basic hypergeometric series is defined as:

$$\begin{aligned} \Phi \left[\begin{matrix} a_1, a_2, \dots, a_A, b_1, b_2, \dots, b_B, \dots; z \\ c_1, c_2, \dots, c_C, d_1, d_2, \dots, d_D, \dots \end{matrix} \right] \\ = \sum_{r=0}^{\infty} \frac{[a_1]_r [a_2]_r \cdots [a_A]_r [b_1]_r [b_2]_r \cdots [b_B]_r \cdots}{[q]_r [c_1]_r [c_2]_r \cdots [c_C]_r [d_1]_r [d_2]_r \cdots [d_D]_r \cdots} z^r \end{aligned}$$

where z may be a function of r , for example, $z^r = q^{r/2(r+1)}$. This notation will be further contracted to $\Phi \left[\begin{matrix} (a), (b), \dots; z \\ (c), (d), \dots \end{matrix} \right]$.

3. An expansion of a generalised basic hypergeometric series in a series of another generalised basic hypergeometric series.

Received November 12, 1962. The author is grateful to Dr. R. P. Agarwal for suggesting the problem and for his helpful guidance throughout the preparation of this paper. The author is also thankful to the Council of Scientific and Industrial Research, Government of India, for giving him financial assistance.