PROTEUS FORMS OF WILD AND TAME ARCS

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1. **Introduction.** For definitions and notations see §2 below.

In the Fox-Artin paper [3] arcs were constructed that failed to be tame because of their shape or form in a small neighborhood of a finite set W. It is easy to extend these examples so that W is denumerable. In this note certain prototypes are found for arcs that fail to be tame because of their shapes near an infinite set of points. One purpose here is to give a method for choosing from a class of equivalent (relative to a space homeomorphism) arcs one of its nicest members. For example, in a given equivalence class an arc J can be chosen so that if J fails to be locally polygonal at x then J fails to be locally tame at x. In this way tedious and frustrating but unnecessary difficulties can be removed in certain situations involving arcs.

The main results of this paper are summarized in the following three theorems. Theorem 1, Theorem 2 and Lemma 7 are combined and stated as Theorem 0.

Theorem 0. *Let A be an arc in E^3. Let $\epsilon > 0$ be given. Then there is a space ϵ-homeomorphism f that maps A onto an arc B, and f and B have the following properties.*

(i) *f is pointwise the identity on the complement of $N(A, \epsilon)$.*
(ii) *B is locally polygonal at each x that is an interior point of some tame subarc of B.*
(iii) *If B fails to be locally polygonal at x, then $x = f(x)$.*
(iv) *If there is a point p on B and a neighborhood $N(p)$ that contains no straight line segment of B, then for an arbitrary plane Q there is a plane P parallel to Q such that $P \cap B \cap N$ has an infinite cardinal but is not a dense subset of any arc in P.*
(v) *Further f may be chosen so that each pair of straight line segments of B (if any) are either parallel or perpendicular.*

Theorem 3. (A characterization of completely wild arcs). *Let A be an arc in E^3. The arc A is completely wild if and only if for each space homeomorphism f and any parametric representation of $f(A)$ each parameter function assumes a local maximum at each point of an ever-where-dense subset of the parameter interval.*

Theorem 4. *Let T, W, and E be the subsets of an arc A such that:* (i) *T is the set at which A is locally tame,* (ii) *$W = A \backslash T$, and* (iii) *E is the subset of W such that each neighborhood of each point in E meets T in a non-empty set. If E lies*

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