# THE DEFICIENCIES OF MEROMORPHIC FUNCTIONS OF FINITE LOWER ORDER

By Albert Edrei

**Introduction.** A few years ago, W. H. J. Fuchs and the present author proved [1] the following

Lemma A. *Let $f(z)$ be a meromorphic function, let $T(r)$ be its Nevanlinna characteristic and let $f(0) = 1$.*

*Denote by*

$$a_1 , a_2 , a_3 , \cdots$$

*the zeros of $f(z)$ and by*

$$b_1 , b_2 , b_3 , \cdots$$

*its poles (multiple values being repeated a suitable number of times).*

*Put*

$$\gamma_0 = 0, \gamma_m = \frac{1}{\pi \rho^m} \int_{-\pi}^{+\pi} \log | f(\rho e^{i\theta}) | e^{-im\theta} \, d\theta \qquad (m \geq 1),$$

*where $\rho(> 0)$ is so small that the disc $|z| \leq \rho$ contains neither zeros nor poles of $f(z)$.*

*Then, if $q$ is a non-negative integer and if*

$$0 \leq r = | z | \leq \tfrac{1}{2}R,$$

*we have*

$$\text{(1)} \qquad \log | f(z) | = \Re\{\gamma_0 + \gamma_1 z + \gamma_2 z^2 + \cdots + \gamma_q z^q\} + \log \Big| \prod_{|a_\mu| \leq R} E(z/a_\mu , q) \Big| - \log \Big| \prod_{|b_\nu| \leq R} E(z/b_\nu , q) \Big| + S_q(z ,R),$$

*where*

$$E(u, 0) = (1 - u); E(u, q) = (1 - u) \exp \{u + \tfrac{1}{2}u^2 + \cdots + (1/q)u^q\} \quad (q \geq 1)$$

*and*

$$| S_q(z, R) | \leq 14 \left\{\frac{r}{R}\right\}^{q+1} T(2R).$$

In a recent paper, Kjellberg [4] has obtained, independently, a special case of Lemma A and has used the result to give an elegant proof of the following theorem. (The special case is characterized by $q = 0$ and $f(z)$ entire.)

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