BROUWER PROPERTY SPACES

By Edwin Duda

1. **Introduction.** In a paper [5] by G. T. Whyburn the following problem was suggested: "How does one characterize those spaces having the property that any subset homeomorphic with an open set is open?". Since Brouwer's Theorem on the Invariance of Domain suggests this problem, it seemed appropriate [2] to say that a space has the Brouwer Property if openness is a topological invariant for subsets of the space. Brouwer's Theorem shows that Euclidean Spaces and manifolds have this property while manifolds with non-empty boundary do not. Also of interest is the converse question [5]: "To what extent are the Euclidean manifolds characterized by this property?". That is, what restrictions are necessary in order to conclude that a space with the Brouwer Property is a manifold of some dimension? We give some answers to both questions and in addition show the existence of non-manifolds with the Brouwer Property in all dimensions.

By mapping we will mean a continuous transformation. For an open subset U of a topological space the frontier of U, denoted by Fr (U), is $\bar{U} - U$. In general definitions which appear in the textbook references are not given. Other terminology is taken directly from the references.

(1.1) *Definition.* A topological space X has the *Brouwer Property* provided that homeomorphic images of open subsets of X are also open subsets of X.

2. **Complexes and the Brouwer Property.** In this section complexes having the B. P. are studied. Results show that complexes of dimension ≤ 2 are necessarily Euclidean manifolds and that this is false for $n \geq 3$. A characterization of such complexes which are locally a Cantorian manifold is given. Simplex here will mean open simplex.

(2.1) *Definitions.* A complex K is a finite set of simplexes in some Euclidean space R^m satisfying the following conditions:

(i) If σ is a simplex of K, then every face of σ is in K.

(ii) For two simplexes σ and σ', their intersection is either empty or a simplex of K.

The dimension of K is the maximum dimension of the simplexes. An n-dimensional complex will be called an n-complex. No distinction is made between the complex K and the topological space determined by K.

(2.2) Lemma. *If K is an n-dimensional complex having the Brouwer Property, then the following are valid:*

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