ALGEBRAICALLY IRREDUCIBLE SEMIGROUPS

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1. **Introduction.** In this note, a semigroup S is a Hausdorff topological space together with a continuous associative multiplication. In particular, a compact connected normal ($Sx = xS$, for all $x \in S$) semigroup S possessing an identity element, denoted by 1, contains a subsemigroup T algebraically irreducible about $K \cup H(1)$ [4] (see definition below), where K is the minimal ideal of S (often called the kernel [6]) and $H(1)$ is the maximal subgroup of S with identity 1 [6]. The concern of this paper is the structure of these algebraically irreducible semigroups. The following definitions are found in [4].

Definitions. (1) A compact connected semigroup S is *algebraically irreducible about* $B \subset S$, if S contains no proper closed connected subsemigroup containing B.

(2) If B consists of two distinct points a and b, S is said to be *algebraically irreducible between* a and b.

The left equivalence of Green [2], defined for a semigroup S by $x \equiv y\mathcal{L}$ if and only if $\{x\} \cup Sx = \{y\} \cup Sy$, will be used as in [3], [4] and [5]. Denote by L_x, the set of all points p such that $p \equiv x(\mathcal{L})$. Since S is compact, the sets L_x form an upper semi-continuous decomposition of S. It has been shown (see [3]) that $\mathcal{L}$ is a congruence for normal semigroups. The quotient space S modulo $\mathcal{L}$ is then a compact semigroup when S is compact and normal and the canonical mapping, denoted by φ, is a continuous homomorphism. Denoting this hyperspace by S', $\varphi: S \to S'$ is given by $\varphi(x) = \{L_x\}$. It was shown in [4] that S' is a standard thread [1] if S is algebraically irreducible about $K \cup H(1)$. In [5], necessary and sufficient conditions that S' be a standard thread were given.

2. **A theorem on inverse limits.** If S is a normal semigroup, then let E denote the set of idempotent elements in S (i.e., $e \in E \leftrightarrow e = e^2$) and $H(e)$ the maximal subgroup of S containing the idempotent element e. The set E is partially ordered by $e < f$ if and only if $ef = e$ (for normal semigroups, note that $e < f$ and $f < e$ imply $e = f$). If e and $f \in E$ with $e < f$, define $\pi_{ef} : H(f) \to H(e)$ by $\pi_{ef}(x) = ex$, then π_{ef} is a continuous homomorphism and $\{H(f), \pi_{ef}\}$ is an inverse system of groups. For $e \in E$, let $E(e) = [f \in E : f < e$ and $e \neq f]$. For S a compact connected normal semigroup algebraically irreducible about $K \cup H(1)$, it is known from [5] that $(E, <)$ is a totally ordered set. Such a semigroup S, that is, a compact, connected, normal semigroup algebraically irreducible about $K \cup H(1)$, will be called an A-I semigroup.

If S is an A-I semigroup and $e \in E$ such that $e \in \overline{E(e)}$ (the bar denotes closure),

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