INTEGRAL OPERATORS IN THE STUDY OF AN ALGEBRA AND OF A COEFFICIENT PROBLEM IN THE THEORY OF THREE-DIMENSIONAL HARMONIC FUNCTIONS

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To the memory of Leo Lichtenstein

Modern function theory as developed by Nevanlinna and others shows that various relations between the growth and value distribution, valid for rational functions, hold (in a modified form) for meromorphic functions. Using the integral operators, we define a composition for the linear space of harmonic functions of *three* variables, so that various sub- and coalgebras of the resulting algebra display similar properties. (Algebra = algebra with respect to the addition and the *composition* to be defined below.)

1. Introduction. Various integral operators **P** transform analytic functions f of one (and several) complex variables into solutions ψ of linear partial differential equation $\mathbf{L}(\psi) = 0$ (see [B.1], [B.2], [B.3], [B.7]). These operators map analytic functions f into solutions ψ at first *in the small*, i.e., f and ψ are considered in a sufficiently small neighborhood $\mathcal{V}(O)$ of the origin O. f is called the associate of ψ with respect to the operator. In many instances by analytic continuation of f and ψ we obtain relations between the associate f (analytic functions of one or several complex variables) and the solutions ψ in the domain of association (i.e., in the domain where the representation of $\psi = \mathbf{P}(f)$ is valid), or *in the large*, i.e., in the whole domain of existence of the solution (see [B.7; 49]).

The solutions of a linear partial differential equation with entire coefficients represent a linear space, while analytic functions form an algebra. Using the integral operator, we can define a composition of solutions ψ which corresponds to the multiplication of the associate functions. In this way we obtain an algebra of solutions of differential equations under consideration, where the ordinary multiplication is replaced by a composition corresponding to the multiplication of the associate functions (see [B.1; 647]).

It is of interest to study solutions ψ which correspond to various algebras (with respect to addition and ordinary multiplication) of associates, such as

1. all analytic functions regular in $\mathcal{V}(O)$,
2. all analytic functions possessing certain singularities at the origin O,
3. rational functions regular at O,
4. meromorphic functions regular at O

(see [B.2], [B.3], [B.8]).

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