DETERMINATION OF AN UNKNOWN COEFFICIENT IN A PARABOLIC DIFFERENTIAL EQUATION

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1. **Introduction.** In [3], B. F. Jones considered the problem of determining the conductivity of a medium if the conductivity was known *a priori* to be a function of time only. Specifically, Jones treated the problem

$$(1.1)\quad \begin{cases} u_t = a(t)u_{xx}, & 0 < x < 1, \quad 0 < t < T, \\ u(0, t) = f_1(t), & 0 < t < T, \quad f_1(0) = 0, \\ u(1, t) = f_2(t), & 0 \le t < T, \quad f_2(0) = 0, \\ u(x, 0) = 0, & 0 \le x \le 1, \\ -a(t) \lim_{x \downarrow 0} u_x(x, t) = g(t), & 0 < t < T, \end{cases}$$

where $a(t)$ is the unknown conductivity. Jones gave conditions on the data $f_1(t)$, $f_2(t)$, and $g(t)$ which enabled him to prove existence and uniqueness of a solution (a pair of functions $u(x, t)$ and $a(t)$ which satisfy (1.1)) of (1.1).

In this article a different approach to the problem of determining the conductivity $a(t)$ is considered. This approach yields a simple analysis of the existence and uniqueness problem. It also yields a numerical technique of approximating $a(t)$ [1]. Consider the problem

$$(1.2)\quad \begin{cases} u_t = a(t)u_{xx}, & 0 < x < 1, \quad 0 < t < T, \\ u(0, t) \equiv \varphi_0, & 0 \le t < T, \\ u(1, t) = \psi(t), & 0 \le t < T, \\ u(x, 0) = f(x), & 0 \le x \le 1, \quad f(0) = \varphi_0, \quad f(1) = \psi(0), \\ a(t) \lim_{x \downarrow 0} u_x(x, t) = h(t), & 0 < t < T, \end{cases}$$

where $\psi(t)$, $f(x)$ and $h(t)$ are known continuous functions of their arguments, and φ_0 is a given constant. From physical experience, the conductivity is assumed to be positive for all time. A solution to (1.2) is defined as follows:

DEFINITION 1.1. A pair of functions $u(x, t)$ and $a(t)$ is a *solution* of (1.2) if and only if the following conditions are satisfied:

(a) $a(t)$ is positive and continuous for $0 \le t < T$;
(b) $u(x, t)$ is continuous in (x, t) for $0 \le x \le 1$, $0 \le t < T$;

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