SIMULTANEOUS REPRESENTATIONS IN QUADRATIC AND LINEAR FORMS OVER GF [q, x]

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1. Introduction. Let $GF(q)$ denote the finite field of order q, where q is odd and let

$$(1.1) \qquad Q(u) = \sum_{i,j=1}^{n} \alpha_{ij} u_i u_j \qquad (\alpha_{ij} \in GF(q))$$

denote a non-singular quadratic form over $GF(q)$. Let $k \geq 1$ and let M be a polynomial in $GF[q, x]$ of degree $<2k$; the case $M = 0$ is allowed. If s is a fixed integer ≥ 1, we let

$$N_s(M) = N_s(M, Q)$$

denote the number of solutions $U_1, \cdots, U_s$ of the equation

$$(1.2) \qquad Q(U_1, \cdots, U_s) = M,$$

where

$$U_j \in GF[q, x], \qquad \deg U_j < k \qquad (j = 1, \cdots, k).$$

Explicit formulas for $N_s(F)$ have been found by Cohen [4], [5]; see also [1], [2]. The formulas depend upon the discriminant of Q and the parity of s.

Let $\delta_i(A)$ denote the number of (primary) divisors of A of degree i; in particular when $A = 0$ we put $\delta_i(0) = q^i$. Put

$$(1.3) \qquad \begin{cases} \gamma_i(A) = \delta_i(A) - \delta_{i-1}(A), \\ \gamma_i'(A) = (q-1)\,\delta_i(A) + \delta_{2k-i}(A) - q\,\delta_{2k-i-1}(A) \end{cases}$$

and define

$$R_t(M, \mu) = \sum_{i=0}^{k} q^{t(2k-i)} \mu^i \gamma_i(M) + \sum_{i=0}^{k-1} q^{t(2k-i)} \mu^i \gamma_i'(M),$$

where μ is an arbitrary complex number. Then if $s = 2t$, α is the discriminant of Q and $\mu = \psi((-1)^t \alpha)$, where

$$(1.4) \qquad \psi(\beta) = \begin{cases} +1 & (\beta \text{ square}) \\ -1 & (\beta \text{ non-square}) \end{cases}$$

then we have

$$(1.5) \qquad N_s(M, Q) = R_{t-1}(M, \mu).$$

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