A PROBLEM IN PARTITIONS

By L. Carlitz

1. We consider partitions of the bipartite (n, m)

$$\begin{cases} n = n_1 + n_2 + n_3 + \cdots \\ m = m_1 + m_2 + m_3 + \cdots , \end{cases} \tag{1.1}$$

where the n_j , m_j are non-negative integers that satisfy the conditions

$$\min (n_j , m_j) \geq \max (n_{j+1} , m_{j+1}) \qquad (j = 1, 2, 3, \cdots). \tag{1.2}$$

For brevity we may write (1.2) in the form

$$(n_j , m_j) \geq (n_{j+1} , m_{j+1}).$$

Let $\pi(n, m)$ denote the number of partitions (1.1) that satisfy (1.2) and let $\pi(n, m \mid a, b)$ denote the number of these partitions that in addition satisfy

$$(a, b) \geq (n_1 , m_1). \tag{1.3}$$

In the latter case we may say that the largest "part" does not exceed (a, b).

Following Chaundy [2] we consider the recurrence

$$\xi_{nm} = \sum_{r,s=0}^{\min (n,m)} x^r y^s \xi_{rs} , \tag{1.4}$$

where ξ_{nm} is a power series in x, y. If we put

$$\xi = \xi_{\infty\infty} , \tag{1.5}$$

then in the limit (1.4) becomes

$$\xi = \sum_{r,s=0}^{\infty} x^r y^s \xi_{rs} . \tag{1.6}$$

It follows from (1.6) and (1.4) that ξ is the generating function of partitions (1.1) that satisfy (1.2); similarly ξ_{ab} is the generating function of these partitions that in addition satisfy (1.3). We may therefore write

$$\xi = \sum_{r,s=0}^{\infty} \pi(r, s) x^r y^s, \tag{1.7}$$

$$\xi_{ab} = \sum_{r,s=0}^{\infty} (r, s \mid a, b) x^r y^s. \tag{1.8}$$

We define the generating functions

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