THE SEGMENTAL VARIATION OF BLASCHKE PRODUCTS

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1. **Introduction.** Let us say that a function which is regular in the open unit disk D has *finite segmental variation* at a point $e^{i\theta}$ (θ real) provided every line segment connecting $e^{i\theta}$ to a point of D is mapped onto a rectifiable curve by the function. If the radius terminating at $e^{i\theta}$ is carried onto a rectifiable curve, we shall use Rudin's terminology [9] and say that the function has *finite radial variation* at $e^{i\theta}$.

Clearly, at a given point $e^{i\theta}$, finite segmental variation entails finite radial variation; and, in turn, finite radial variation entails the existence of a (finite) radial limit.

Seidel and Walsh [10; 143] proved that, if a function is regular and univalent in D, it has finite segmental variation at almost all points of the unit circumference C.

Beurling [1] proved that, if a function is regular in D and has a finite Dirichlet integral, then it has finite radial variation at each point of C except on a set whose outer capacity is zero. Subsequently, Tsuji (see [11; 344]) showed that "finite radial variation" may be replaced by "finite segmental variation" in the statement of Beurling's theorem.

In this paper, we shall consider a well-known class of functions whose membees are not univalent and do not have finite Dirichlet integrals, namely, (infinitr) Blaschke products.

We remind the reader that a sequence $\{z_n\}$ of complex numbers satisfying the conditions

$$(1.1) \qquad 0 < |z_n| < 1 \quad \text{and} \quad \sum_n (1 - |z_n|) < \infty$$

is called a *Blaschke sequence* and that the associated function of the form

$$(1.2) \qquad B(z; \{z_n\}) = \prod_n b(z; z_n)$$

where

$$(1.3) \qquad b(z; z_n) = \frac{|z_n|}{z_n} \frac{z_n - z}{1 - \bar{z}_n z}$$

is called a *Blaschke product*. (For a general discussion of Blaschke products, see [7; 49–52] and [12; 271–285].) Throughout this paper, we shall, for obvious reasons, consider only infinite Blaschke products, that is, Blaschke products whose associated Blaschke sequences have infinitely many elements.

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