A NOTE ON A PAPER OF GINSBURG

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Introduction. Let P be a partially ordered system and let S and T be non-empty subsets of P. If, for every $p \varepsilon S$, there exists a $q \varepsilon T$ such that $q \geq p$, T is said to be confinal in S. For every $p \varepsilon P$, we denote the set of successors of p in P by $A_P(p)$. If two partially ordered systems P and Q are order isomorphic with confinal subsets of some partially ordered system, they are said to be confinally similar. A partially ordered system P without maximal elements is said to have sufficiently many non-cofinal subsets if, for any two distinct elements p and q of P, either $A_P(p)$ is not cofinal in $A_P(q)$ or $A_P(q)$ is not cofinal in $A_P(p)$. The properties of sets having sufficiently many non-cofinal subsets have been investigated by Ginsburg [1], who poses the following question: "If P has sufficiently many non-cofinal subsets and Q is cofinally similar to P, does Q contain a cofinal subset S which has sufficiently many non-cofinal subsets?" It will be shown by example that the answer to this question is negative.

A subset S of a partially ordered system P is said to be a residual subset if, for every $p \varepsilon S$, $A_P(p)$ is contained in S. A subset S of P is said to be maximal residual if S is a residual subset which is not a proper cofinal subset of any residual subset of P. The set of maximal residual subsets of P, ordered by the dual of set inclusion, is denoted by $F(P)$. Ginsburg proves the following theorem (Theorem 5 of [1]): *If P has sufficiently many non-cofinal subsets, P is cofinally similar to $F(P)$.* It is shown that the proof given for this theorem is invalid, and a counterexample to the theorem is given.

1. An example. An example is to be given of two cofinally similar partially ordered systems, one of which has sufficiently many non-cofinal subsets and the other of which contains no cofinal subset having sufficiently many non-cofinal subsets.

Let ω_1 be the first non-denumerable ordinal, and let $W(\omega_1)$ be the set of ordinals less than ω_1. Associate with each $x \varepsilon W(\omega_1)$ an infinite subset A_x of the set of integers in such a way that distinct ordinals are assigned distinct sets of integers. Now, for any finite set of integers A, one of the following two cases occurs:

i. For each $x \varepsilon W(\omega_1)$ there exists an $s \varepsilon W(\omega_1)$, $s \geq x$, such that $A \subset A_s$.

ii. For some $x \varepsilon W(\omega_1)$, A is not contained in A_s for any $s \geq x$, while, for all $y < x$, there exists a $z \varepsilon W(\omega_1)$, $z \geq y$, such that $A \subset A_z$.

We now consider the set of all x's associated with sets of integers in the second category. This is a denumerable set of denumerable ordinals; hence, there exists a denumerable ordinal ω' which is greater than any of the ordinals

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