QUASI-TOPOLOGIES

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1. **Introduction.** If X and Y are topological spaces and $C(X; Y)$ denotes the set of continuous mappings from X to Y, it is reasonable to try to topologize $C(X; Y)$ so that one or more natural conditions are satisfied. One such condition on a topology on $C(X; Y)$ is that if Z is a topological space then a map from Z into $C(X; Y)$ is continuous (in the topology on $C(X; Y)$ in question) if and only if the corresponding map of $Z \times X$ into Y is continuous (from the product topology on $Z \times X$). If X is a locally compact Hausdorff space, then the compact-open topology on $C(X; Y)$ [4; 7] (called the k-topology in [1]) has this property; however if X is a completely regular space which is not locally compact and Y is the unit interval, it follows from [1, Theorem 3] that there is no topology on $C(X; Y)$ with the above natural property (for details see (5.2) of the appendix of the present note). Even if one relaxes the natural property above so that instead of requiring it to be valid for arbitrary topological spaces Z it is just required for compact Hausdorff spaces, there need not be a topology on $C(X; Y)$ with this property. Lemma (5.5) of the appendix furnishes a counterexample.

In order to retain the above natural property on $C(X; Y)$ the concept of a quasi-topology was introduced [3; 5; 9; 10] and $C(X; Y)$ was provided with a quasi-topology having the natural property even when it couldn't be provided with a topology having the property. A quasi-topology on a set is a collection of mappings from certain topological spaces into the set satisfying a few simple conditions. If the set has a topological structure on it, then the collection of continuous mappings into it is a quasi-topology on the set, but there are quasi-topologies on a set which do not arise in this way from any topology on the set (see the note following (5.5) of the appendix).

In the definition of quasi-topology it is possible (and useful) to restrict the class of topological spaces being mapped into the set rather than to allow arbitrary spaces as in [9]. In the present article we shall define a quasi-topology on a set by means of mappings from compact Hausdorff spaces into the set. Doing so makes it possible to connect up the weak topology $\langle X \rangle$ defined by the compact subsets of a Hausdorff space X considered in [8, §2] with the concept of a quasi-topology.

This article is devoted to a definition of quasi-topology and induced quasi-topologies and to a study of some of their basic properties. It is shown how the usual constructions of homotopy theory such as suspensions and loop spaces can be carried out for quasi-topological spaces in such a way that the standard properties remain valid. In particular, if X and Y are topological spaces with

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