SOME GLOBAL PROPERTIES OF THE SPACE OF HOMEOMORPHISMS ON A DISC WITH HOLES

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In [6], Eldon Dyer and I proved that the space of homeomorphisms of a 2-manifold with boundary onto itself that leave the boundary pointwise fixed is locally contractible. A well-known result of Alexander's [1] is that the space of homeomorphisms of an n-cell onto itself leaving the boundary pointwise fixed is contractible and locally contractible. In [6], it was also proved that the space of homeomorphisms of an annulus onto itself leaving one of its boundary curves pointwise fixed is contractible. The proof of this fact also demonstrates that the identity component of the space of homeomorphisms of an annulus onto itself leaving its boundary pointwise fixed is contractible. The purpose of the present note is to show that the identity component of the space of homeomorphisms of a disc with holes onto itself that leave the boundary pointwise fixed is homotopically trivial. In a later paper, some of these results will be applied to 2-manifolds with boundary in general. For related results the reader is referred to the other references listed at the end of this paper.

A mapping z of a metric space X onto a metric space Y is said to be *completely regular* if for each positive number ϵ, there is a positive number δ such that $d(y, y') < \delta$, $y, y' \in Y$, implies that there is a homeomorphism of $z^{-1}(y)$ onto $z^{-1}(y')$ that moves no point as much as ϵ (i.e., an ϵ-homeomorphism). In [5] were proved several theorems that give conditions which imply that (X, z, Y) is a locally trivial fiber space or a direct product. Here is another such condition, easily deducible from arguments in [5].

LEMMA. *Let K be a compact metric space and L a closed subset of K such that the space of homeomorphisms of K onto itself leaving L pointwise fixed is locally connected. Suppose further that z is a completely regular mapping of a complete metric space X onto an n-cell Y such that for each point y of Y, there is a homeomorphism z_y of K onto $z^{-1}(y)$. Suppose, further, that there is a homeomorphism h of $\bigcup z_y(L)$ onto $Y \times L$ such that the diagram*

$$\begin{array}{ccc} \bigcup z_y(L) & \xrightarrow{\quad h \quad} & Y \times L \\ & {\scriptstyle z} \searrow & \big\downarrow {\scriptstyle \pi} \\ & & Y, \end{array}$$

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