FIXED POINTS OF CONTINUOUS MAPPINGS INTO EUCLIDEAN n-SPACE

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In this paper the authors prove that if the fixed points of a continuous mapping of an oriented n-complex into its embedding Euclidean n-space are isolated and solely at inner points of n-cells of the complex, then the algebraic sum of the orders (defined below) of the fixed points is equal to the turning index (defined below) of the boundary of the complex.

In what follows, a Euclidean n-space R^n with an assigned orientation is assumed to be fixed once and for all and the same orientation, called positive, is assigned to any rectilinear n-simplex or (polyhedral) n-cell or any n-ball, and the induced orientations of their boundaries are called the positive orientations of these boundaries.

Also, D^{n-1} denotes a positively oriented $(n - 1)$-sphere with center 0, located in an arbitrary position in R^n, which serves as a *direction sphere* [3].

Let f be a continuous mapping of an oriented n-complex K^n in R^n whose n-cells are also oriented. Let C^{n-1} be any oriented $(n - 1)$-cycle on K^n, such that no point of C^{n-1} is fixed under f, and let ϕ be the continuous mapping of C^{n-1} into D^{n-1} defined as follows. For every $c \in C^{n-1}$ the direction from 0 to $\phi(c)$ is the same as that of c to $f(c)$. The degree of the mapping of ϕ on D^{n-1} (i.e., the multiple of D^{n-1} which is homologous on D^{n-1} to $\phi(C^{n-1})$) will be called the *turning index* of C^{n-1} under f. It is clearly independent of the location and the size of D^{n-1}. Also, in this connection, $\phi(C^{n-1})$ is called *the $(n - 1)$-cycle on D^{n-1} resulting from f applied to C^{n-1}*.

If C^{n-1} is an oriented $(n - 1)$-cycle on K^n and e is a point of R^n not on C^{n-1}, we let ψ be the map from C^{n-1} into the direction sphere such that if $c \in C^{n-1}$; then the direction from e to c is the same as from 0 to $\psi(c)$. The degree of the resulting map is called the *index* of e relative to C^{n-1}. (Cf. [5; 327]). It is clear that if two points can be joined by a path on K^n not meeting C^{n-1}, their indices relative to C^{n-1} are equal.

Let, as before, f be a continuous mapping of K^n into R^n and p be an *isolated fixed point* under f, i.e., there exists an $(n - 1)$-sphere V^{n-1} with center at p such that V^{n-1} and its interior are in an n-cell of K^n containing p and that there is no fixed point other than p on or inside V^{n-1}. (By the invariance of regionality, for sufficiently small V^{n-1} its interior will consist of points of the n-cell.) Then the turning index of V^{n-1} under f is independent of the size of V^{n-1} and it is called the *order* of the isolated fixed point p under f (Cf. [5; 327]).

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