A COROLLARY OF THE GOLDBACH CONJECTURE

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1. **Introduction.** The far-famed conjecture of Goldbach asserts (in a variant form) that every sufficiently large even integer is representable as a sum of two distinct odd primes. A non-trivial consequence of this assertion is contained in the following result:

Theorem 1. *Corresponding to every sufficiently large even integer n, there exist odd primes p_1, p_2, both less than $n - 1$, such that $n - p_1$ and $n - p_2$ are relatively prime.*

This result may be stated alternatively as follows. For an arbitrary positive integer n, let $E(n)$ denote the number of sets of integral solutions $\{x_1, x_2, p_1, p_2\}$ of

$$n = x_1 + p_1, \quad n = x_2 + p_2, \quad x_1 > 1, \quad x_2 > 1, \tag{1.1}$$
$$(x_1, x_2) = 1, \quad p_1, p_2 \text{ odd primes};$$

then $E(n) > 0$ for every sufficiently large even integer n.

Theorem 1 is a consequence of the much stronger result,

Theorem 2.

$$\lim_{n\to\infty} E(n) = \infty \qquad (n \text{ even}). \tag{1.2}$$

In this note we prove an asymptotic formula for $E(n)$ which yields Theorem 2 as an almost immediate corollary. This result is contained in Theorem 3 below. Except for appropriate modifications and some slight simplifications, the method used is the same as that employed by Estermann in his treatment [3] of the number of representations of an integer as a sum of a prime and a square-free integer. The two main tools required in the proof are the classical estimate for the number of primes in an arithmetical progression (Lemma 1) and a special case of the Brun-Titchmarsh Theorem (Lemma 3). In view of A. Selberg's elementary proof of the first of these results [5], the method of proof of the paper can be considered (technically) elementary.

In another paper [1], a refinement of our main result is proved in a much more general setting. The proof is not, however, elementary, as it is based upon a deep corollary [2] of the Page-Siegel-Walfisz Theorem.

2. **Preliminaries.** Let **x** denote a positive number and **n**, **k**, **l** positive integers. Let $\phi(n)$ and $\mu(n)$ denote the Euler and Möbius functions, respectively, $\pi(x)$

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