THE STRUCTURE OF SOME INDUCED REPRESENTATIONS

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Introduction. Let H be a normal subgroup of the discrete group G and let L be a finite dimensional irreducible unitary representation of H. In this paper we give a partial description of the primary components of the induced representation U^L. The most commonly met example of such a representation is the regular representation and, not unexpectedly, all representations induced from a normal subgroup behave very much like the regular representation. More precisely, we show (Theorem 7) that the commutant of U^L is isomorphic to the commutant of a regular projective representation of a certain subgroup of G/H. Thus, to a large extent, the study of representations induced from a normal subgroup reduces to the study of regular projective representations. For this reason we devote the first part of this paper to a description of regular projective representations. These representations also behave very much like ordinary regular representations, the major difference lying in the role played by a distinguished subset of the set of all conjugate classes in G. But once this fact has been noticed, many of the known results about ordinary regular representations can be reproduced in the setting of projective regular representations. In the second part we turn to the study of U^L. Here the essential result is Theorem 7 which enables us to carry over the results of the first part.

We have benefited from illuminating conversations on this subject with Professor G. W. Mackey. The proof of the equality iv is due to the referee.

1. Regular projective representations.

1.1. The commutant of a regular projective representation.

Let G be a discrete group. A multiplier ω on G is a function on $G \times G$ with values in the group of complex numbers of modulus one with the properties:

i. $\omega(x, e) = \omega(e, x) = 1, \quad \text{all} \quad x \in G$

ii. $\omega(x, y)\omega(xy, z) = \omega(x, yz)\omega(y, z), \quad \text{all} \quad x, y, z \in G \times G \times G.$

If ω has the further property:

iii. $\omega(x, x^{-1}) = 1, \quad \text{all} \quad x \in G,$

we shall say that ω is normalized. Let ρ be a function on G with values in the group of complex numbers of modulus one such that $\rho(e) = 1$. The function $\omega' : x, y \longrightarrow \rho(x)\rho(y)\rho(xy)^{-1}\omega(x, y)$ is also a multiplier on G when ω is and ω and ω' are said to be similar. Every multiplier ω on G is always similar to a normalized multiplier ω'. Indeed, we may take $\omega'(x, y) = (\omega(x, x^{-1})\omega(y, y^{-1}))^{-\frac{1}{2}} \omega(xy, y^{-1}x^{-1})^{\frac{1}{2}}\omega(x, y)$. An ω representation of G (more precisely, a unitary ω representation) is a map L of G into the group of unitary operators of a Hilbert

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