SINGULAR PERTURBATIONS OF A BOUNDARY VALUE PROBLEM FOR A NONLINEAR SYSTEM OF DIFFERENTIAL EQUATIONS

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1. **Introduction.** This paper is concerned with the existence and asymptotic character of solutions of the nonlinear boundary value problem

$$\begin{aligned} \frac{dx}{dt} &= f_1(t, x, \epsilon) + f_2(t, x, \epsilon)y \\ \epsilon \frac{dy}{dt} &= g_1(t, x, \epsilon) + g_2(t, x, \epsilon)y \end{aligned} \tag{1.1}$$

$$x(\alpha, \epsilon) = l_1 , \qquad x(\beta, \epsilon) = l_2 \tag{1.2}$$

as $\epsilon \rightarrow 0+$ when the *degenerate* differential system

$$\begin{aligned} \frac{dx}{dt} &= f_1(t, x, 0) + f_2(t, x, 0)y \\ 0 &= g_1(t, x, 0) + g_2(t, x, 0)y \end{aligned} \tag{1.3}$$

has a solution $x = \varphi(t)$, $y = \psi(t)$ which satisfies one of the boundary conditions (1.2).

We shall make the following assumptions.

H1: *The degenerate differential system* (1.3) *possesses a solution,* $x = \varphi(t)$, $y = \psi(t)$ *of class* $C^{(\infty)}$ *for which*

$$\varphi(\beta) = l_2 \quad and \quad g_2(t, \varphi(t), 0) < 0, \qquad \alpha \leq t \leq \beta. \tag{1.4}$$

H2: *The functions* f_1, f_2, g_1, g_2 *are regular analytic with respect to* x *and* ϵ *and of class* $C^{(\infty)}$ *with respect to* t *in a region* R *of* x, t, ϵ*-space that contains in its interior all points,* $x = \varphi(t)$, $\alpha \leq t \leq \beta$, $\epsilon = 0$.

Without loss of generality, we may assume that

$$\begin{cases} \alpha = 0, \qquad \beta = 1, \\ \varphi(t) \equiv 0, \qquad \psi(t) \equiv 0, \\ g_2(t, 0, 0) = -1, \\ f_1(t, 0, 0) = g_1(t, 0, 0) = \dfrac{\partial}{\partial x} g_1(t, 0, 0) = 0, \end{cases} \tag{1.5}$$

since the transformation of variables,

Received July 17, 1961. Sponsored by the Mathematics Research Center, U.S. Army, Madison, Wisconsin, under Contract No. DA-11-022-ORD-2059.