THE HOMOLOGY OF DELETED PRODUCTS OF TREES

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1. **Introduction.** If X is a space and $k > 1$, then the k-th deleted product space, X_k^*, of X is the topological product $X \times X \times \cdots \times X$ of k copies of X minus the set of all points of the form $(x, x, \cdots, x)$, where $x \in X$. The objective of this paper is to show that the groups $H_m(A_k^*, Z)$, where A is a tree (finite, contractible, 1-dimensional polyhedron) and Z is the group of integers, do not provide any new information over and above the classical numerical invariants x_i = number of vertices of order i in A. We show that two trees, A and B, have the same number of vertices of the same order i, for each $i > 2$, if and only if $H_m(A_k^*, Z)$ is isomorphic to $H_m(B_k^*, Z)$ for each m and k. Therefore we have reduced the unsolved problem of topological classification of trees by means of algebraic invariants to the problem of classification of trees which have the same number of vertices of the same order by means of algebraic invariants. The usefulness of the groups $H_m(A_k^*, Z)$ in distinguishing spaces of the same homotopy type has been demonstrated by Hu ([1]) and the author ([2]). Hu also showed that the Euler characteristic $\chi(A_k^*)$ does not provide more information than the numbers x_i. The methods used in computing $H_m(A_k^*, Z)$ in this paper are, in part, extensions of those methods used by Hu.

In Section 3, we show that, for each m and k, $H_m(A_k^*, Z)$ is a free Abelian group, and we compute the exact number of generators of each of these groups in terms of the orders of the vertices of A. (The author ([2]) has previously computed $H_m(A_2^*, Z)$.)

In §4, we show that these groups distinguish trees which do not have the same number of vertices of a given order.

If X and Y are spaces, we use the notation, $X \equiv Y$, to mean that X is homeomorphic to Y. The number of combinations of k things taken i at a time is denoted by $C(k, i)$. The group $H_0(K, Z)$ is the direct sum of $m - 1$ groups each isomorphic to Z, where m denotes the number of components of K.

2. **Some preliminary theorems.**

DEFINITION 2.1. If X is a topological space and $k > 1$, then the *diagonal*, D_X^k, of the topological product $X \times X \times \cdots \times X$ of k copies of X is the set of all points of the form $(x, x, \cdots, x)$, where $x \in X$.

DEFINITION 2.2. If X is a topological space and $k > 1$, then the k-th *deleted product space*, X_k^*, of X is the space $X_1 \times X_2 \times \cdots \times X_k - D_X^k$, where each $X_i = X$, with the relative topology.

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