INTEGRAL GEOMETRY IN THE MINKOWSKI PLANE

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Blaschke, in [2], introduces densities for geodesics and geodesic segments in a two-dimensional Finsler space S and uses these densities to generalize some of the formulas of Euclidean integral geometry. In this paper we consider the special case where S is a Minkowski plane, and, using the apparatus developed by Biberstein, [1], in his elegant treatment of the Minkowskian geometry, we generalize some more of the results of Euclidean integral geometry—see equations (4.7), (5.5), and (5.9). The reader will see that by use of this formalism a great many of the Euclidean results are carried over quite simply to the Minkowski plane.

The author is deeply indebted to Professor István Fáry for many valuable discussions along these lines.

1. **Introductory concepts.** Let U be a centrally symmetric closed convex curve enclosing area π and with center at the origin $\mathbf{0}$ of the Euclidean plane R^2. We shall assume throughout that U is "sufficiently" differentiable and has positive finite curvature everywhere.

The *Minkowskian distance*, $|\mathbf{x} - \mathbf{y}|$, from $\mathbf{x}$ to $\mathbf{y}$ is defined by

$$|\mathbf{x} - \mathbf{y}| = r^{-1} |\mathbf{x} - \mathbf{y}|_e , \tag{1.1}$$

where $|\mathbf{x} - \mathbf{y}|_e$ is the Euclidean distance from $\mathbf{x}$ to $\mathbf{y}$, and r is the radius of U in the direction of $\mathbf{x} - \mathbf{y}$. The points of R^2, together with this new metric, shall be referred to as the *Minkowskian plane*, M^2.

Following Biberstein, [1], we parametrize U by twice its sectorial area, ϕ, and write the equation of U as

$$\mathbf{t} = \mathbf{t}(\phi), \qquad 0 \leq \phi \leq 2\pi, \qquad |\mathbf{t}| = |\mathbf{t} - \mathbf{0}| = 1. \tag{1.2}$$

U shall be referred to as the *indicatrix*.

We define $\mathbf{n}(\phi)$ by

$$\mathbf{n}(\phi) = \frac{d\mathbf{t}(\phi)}{d\phi}, \qquad 0 \leq \phi \leq 2\pi. \tag{1.3}$$

The *isoperimetrix*, T, introduced by Busemann (see [4]), is the curve with equation

$$\mathbf{n} = \mathbf{n}(\phi), \qquad 0 \leq \phi \leq 2\pi. \tag{1.4}$$

It is easy to verify that T is the polar reciprocal of U, with respect to the Euclidean unit circle, rotated through 90°. We shall always denote by c the area enclosed by T.

Received June 28, 1961; in revised form, October 30, 1961.