CHARACTERS AND CROSS SECTIONS FOR CERTAIN SEMIGROUPS

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1. Introduction. It is our purpose here to consider several aspects of the structure of compact connected semigroups. Our study was originally motivated by the character semigroups of such semigroups. In considering these, we shall have occasion to consider various related topics which arise naturally and are, moreover, of interest in themselves. Among these are the existence of arcs and local threads—particularly at the identity, the existence of various local Cartesian products involving the maximal subgroups, the structure of arc-components, and certain homomorphisms between these subgroups.

We shall make use of the left equivalence of Green [6], defined for a semigroup S. We recall; $x \equiv y(\mathcal{L}) \Leftrightarrow x \cup Sx = y \cup Sy$. We denote by L_x, the set of all points p such that $x \equiv p(\mathcal{L})$. If S is compact then, as is well known, the sets L_x form an upper semi-continuous decomposition of S. In general, the equivalence is not a congruence. However, if S is Abelian, or, more generally, is normal, that is to say $xS = Sx$ for each $x \in S$, then $\mathcal{L}$ is a congruence. Thus, if S is compact and normal, the quotient space S mod $\mathcal{L}$ is again a compact semigroup and the canonical mapping which we denote by ϕ, is a continuous homomorphism. Let us denote this hyperspace by S'.

Consider now, the upper semi-continuous decomposition of S, (normal and compact), whose elements are the components of the sets L_x. Let S'' be the hyperspace. Then S'', with the natural multiplication, is a compact semigroup. The following diagram is commutative:

$$\begin{tikzcd} S \arrow[rr, "\phi"] \arrow[dr, "\delta"'] & & S' \\ & S'' \arrow[ur, "\beta"'] & \end{tikzcd}$$

Of course, $\phi = \beta\delta$ is the well-known factorization of ϕ into monotone and light homomorphisms.

It is fundamental in most of what follows that if S'' is connected, it contains a standard thread from zero to identity. Roughly speaking then, there is an arc of components of sets L_x from L_e to C_1 where C_1 is the component of 1 in L_1. (See [8] and [9].)

It is easy to see that if S is normal and $f^2 = f$, then L_f coincides with H_f being the maximal subgroup containing f. In particular C_f, the component of f in L_f, is a compact connected (topological) group.

Throughout we shall denote the set of idempotents by E.

Let D be a semigroup and f an idempotent in D. We denote by H_f, as is

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