HOMOTOPY GROUPS OF CONFIGURATION SPACES AND THE STRING PROBLEM OF DIRAC

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1. **Introduction.** 1.1. We recall first the definition of configuration space as given in [3]. Let X denote a topological space and Q_m a fixed set of m distinct points of X. The configuration space $F_{m,n}(X)$ is the space of n-tuples $(p_1, \cdots, p_n)$ of mutually distinct points of X with the further condition that no p_i belongs to Q_m, $i = 1, \cdots, n$. Homotopy groups of configuration spaces of various spaces play a useful role in many connections, and the first useful result is due to L. Neuwirth [3], namely,

$$\pi_i(F_{0,n}(E^r)) = \sum_{k=1}^{n-1} \pi_i(\underbrace{S^{r-1} \vee \cdots \vee S^{r-1}}_{k}), \qquad i \geq 2.$$

There are also contained in [3] general results which give $\pi_i(F_{0,n}(X))$ for odd dimensional spheres and for compact 2-manifolds different from the 2-sphere and projective plane. For example, if S^r is an odd sphere

$$\pi_i(F_{0,n}(S^r)) = \pi_i(S^r) + \sum_{k=1}^{n-2} \pi_i(\underbrace{S^{r-1} \vee \cdots \vee S^{r-1}}_{k}), \qquad i \geq 2.$$

The case of even spheres is conspicuously absent from [3], as is the case of projective spaces. In this note we remedy this situation by proving the following theorems, where we assume throughout that $r \geq 2$ since the case $r = 1$ is of little interest.

THEOREM 1. *If S^r is an r-sphere, then for $n \geq 4$*

$$\pi_i(F_{0,n}(S^r)) = \pi_i(V_{r+1,2}) + \sum_{k=2}^{n-2} \pi_i(\underbrace{S^{r-1} \vee \cdots \vee S^{r-1}}_{k}), \qquad i \geq 2.$$

where $V_{r+1,2}$ is the Stiefel manifold [6] *of orthogonal two frames in Euclidean $(r + 1)$-space. For $n = 2, 3$*

$$\pi_i(F_{0,2}(S^r)) = \pi_i(S^r), \qquad i \geq 1.$$

$$\pi_i(F_{0,3}(S^r)) = \pi_i(V_{r+1,2}), \qquad i \geq 1.$$

THEOREM 2. *If P^r is real projective r-space, then for $n \geq 3$*

$$\pi_i(F_{0,n}(P^r)) = \pi_i(V_{r+1,2}) + \sum_{k=1}^{n-2} \pi_i(\underbrace{S^{r-1} \vee \cdots \vee S^{r-1}}_{2k+1}), \qquad i \geq 2.$$

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