SPECIAL HOMEOMORPHISMS IN THE FUNCTIONAL SPACE $\mathcal{C}(X, I_{2n+1})$

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1. **Introduction.** In the following, all spaces are separable metric unless otherwise indicated, and mapping means continuous function. Euclidean n-dimensional space ($n \geq 1$) will be represented by E_n. The cube in E_n consisting of those points in E_n each of whose coordinates x_i satisfies $| x_i | \leq 1$ will be represented by I_n. A k-dimensional hyperplane (k-hyperplane) in E_n (respectively I_n) is a translation of a k-dimensional linear subspace of E_n (respectively such translation intersected with I_n), for $0 \leq k \leq n$.

For compact spaces we have the following known result:

Theorem 1. *Suppose M is a compact metric space of dimension $\leq n$, and $\mathcal{C}(M, I_{2n+1})$ is the space of all mappings of M into I_{2n+1}, equipped with the metric $d(f, g) = \sup \{d(f(x), g(x)) \mid x \in M\}$. If $T_1, T_2, \cdots$ is any countable sequence of n-hyperplanes in I_{2n+1}, then there exists a dense G_δ set $\mathfrak{F} \subseteq \mathcal{C}(M, I_{2n+1})$ such that for all $f \in \mathfrak{F}$ we have:*

(i) *f is a homeomorphism of M into I_{2n+1},*
(ii) *for all r, $f(M) \cap T_r = \phi$ (the vacuous set), and*
(iii) *if $n + 1 \leq k \leq 2n + 1$ and T^k is any k-hyperplane in I_{2n+1}, then $\dim (f(M) \cap T^k) \leq k - n - 1$. Furthermore, every function satisfying* (i)–(iii) *is in $\mathfrak{F}$.*

J. H. Roberts' paper [8] proves (but the result is not explicitly stated) that those mappings f satisfying (i) and (iii) constitute a dense G_δ set $\mathfrak{F}'$. By the argument in [5], proof of Theorem V5, first paragraph, there is a dense G_δ set $\mathfrak{F}''$ whose elements have property (ii). Then $\mathfrak{F}' \cap \mathfrak{F}''$ satisfies the requirements of the set $\mathfrak{F}$ of the above theorem.

The object of the present paper is to extend this result (slightly modified) to non-compact, separable metric spaces (Theorem 3) and to make an application of the new result to generalize a theorem of M. K. Fort, Jr. (See §4. Corollary 1).

2. Now Hurewicz [4] has shown that any separable metric space M has a compactification M^* of the same dimension, but this does not make our problem trivial. For if M^* is any particular compactification of M, there are mappings $f \in \mathcal{C}(M, I_{2n+1})$ which do not have extensions in $\mathcal{C}(M^*, I_{2n+1})$. The principal tool in the proof of the extended result is the following theorem of A. B. Forge [1]:

Theorem 2. *If M is a separable metric space and f: $M \to I_n$ is a mapping,*

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