LIGHT OPEN MAPS ON n-MANIFOLDS. II

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Introduction. This paper continues an investigation of light interior (i.e., open) maps on n-manifolds begun in [6]. The dearth of examples proved a hindrance in this study, and thus the first three sections are largely devoted to constructions. Given $f : M \to N$, the branch set of f, denoted by B_f, is the set of points of M at which f fails to be a local homeomorphism. In both the previous paper and in this one, interest is centered on the nature of branch sets.

Simplicial interior maps are of particular interest for two reasons: first, they are simple; and, second, one hopes that there is a simplicial approximation theorem for light interior maps, the simplicial maps also being interior. The simplicial interior maps f on the 2-sphere contrast sharply with those on the n-sphere, S^n, for $n \geq 3$. For the former, the number of components of B_f is limited by the degree of f; for the latter, it is not. For the former, B_f contains a 0-sphere; for the latter, at least for $n \geq 4$, B_f does not necessarily contain any compact (n-2)-manifold. For the former, B_f is (trivially) the minimal carrier of an (n-2)-cycle; for the latter, it is not necessarily, except mod 2. Also, for $n = 3$, B_f may consist of a knotted simple closed curve.

The general light interior maps on S^n, $n \geq 3$, also contrast with those on S^2 (which, by [16; 121], are topologically equivalent to meromorphic functions). In particular, (for $n \geq 3$), B_f need not be locally connected; indeed, while B_f cannot have *isolated* point components [6; 535, 5.6], it can have point components.

In §4, there is a study of the nature of B_f near points of local degree two; this situation contrasts with the general one in that much of the possible pathology is ruled out. The spaces of meromorphic and light open maps on the 2-sphere are studied in §5.

Each space discussed in this paper is metric, and *M and N are always n-manifolds*. The interior of a space X is denoted by Int (X), and the distance between two points p and q is denoted by $\rho(p, q)$. The terms defined in [6; 528 and 529] are also assumed known.

1. Preliminary definitions and remarks.

Definition 1. (Whyburn [19; 997]). Let $g : A \to B$. For any y in B, let $k(y)$ be the number of points in $f^{-1}(y)$, if this number is finite, otherwise, let $k(y) = \infty$. In case $k(y)$ has a finite maximum d, the *degree* of g is defined to be d.

Definition 2. Let $g : A \to B$ and $p \in A$. For each n ($n = 1, 2, \cdots$)

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