PERMUTATIONS OF TAYLOR COEFFICIENTS OF BOUNDED FUNCTIONS

BY WALTER RUDIN

1. **Introduction.** In this paper, α will always denote a function whose domain of definition is a set Y of non-negative integers and whose values are non-negative integers. To avoid trivialities, Y will always be assumed to be an infinite set. Every such α induces a linear transformation Ψ in the set of all formal power series: if

$$f(z) = \sum_{m=0}^{\infty} c(m)z^m, \tag{1}$$

Ψf is defined by

$$(\Psi f)(z) = \sum_{m \varepsilon Y} c(\alpha(m))z^m. \tag{2}$$

If Q and R are two classes of power series, and if $\Psi f \varepsilon R$ for all $f \varepsilon Q$, we say that α *carries* Q to R. If α carries Q to Q, we say that α preserves Q.

The main result of this paper is a complete characterization of those functions α which preserve the class of all bounded analytic functions in the open unit disc. Before stating this result, it may be of some interest to point out that it is often quite easy to determine the functions α which preserve a class which is characterized by conditions which bear only on the absolute values of the coefficients. Let us illustrate this with a few examples; the proofs are so elementary that we omit them:

(a) Let P_r be the class of all power series whose radius of convergence is at least r. Let D^* and D_* be the upper and lower limits, respectively, of $\alpha(m)/m$, as $m \to \infty$ in Y. Then

(i) α preserves P_∞ if and only if $D_* > 0$;
(ii) for $1 < r < \infty$, α preserves P_r if and only if $D_* \geq 1$;
(iii) α preserves P_1 if and only if $D^* < \infty$;
(iv) for $0 < r < 1$, α preserves P_r if and only if $D^* \leq 1$.

(b) Let H^2 be the class of all power series with $\sum | c(m) |^2 < \infty$; α preserves H^2 if and only if there is an integer k such that no integer has more than k inverse images under α. The same result holds for all functions with finite Dirichlet integral in $| z | < 1$, and for the class of all power series which converge absolutely on $| z | = 1$.

It is convenient to make the following definition: If S is an infinite arithmetic

Received April 4, 1961. Sponsored by the United States Army under Contract No. DA-11-022-ORD-2059. Mathematics Research Center, United States Army, Madison, Wisconsin, and by NSF Grant G-14362.