THE STRUCTURE OF DECOMPOSABLE SNAKELIKE CONTINUA

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The notion of a chainable continuum was introduced by R. L. Moore in [10] (see also [11]); and, in [12], J. H. Roberts showed that any chainable plane continuum has uncountably many disjoint images in the plane. More recently, Moise and Bing used a certain chainable continuum, the pseudo arc, as an example to settle two questions of considerable interest, ([3] and [9]). In [5] Bing defines the notion of a snakelike continuum and shows that it is equivalent to the earlier notion of chainable plane continuum and makes a study of some of the properties of snakelike continua. Recently, Henderson, [7], makes use of the structure of decomposable snakelike continuum to obtain the result that the only decomposable continuum homeomorphic to each of its proper subcontinua is the arc. However, all but one of the fifteen theorems in his paper contain the hypothesis that the set considered is (by his result) an arc.

The purpose of the present paper is to give a necessary and sufficient condition that a snakelike continuum be decomposable (or indecomposable) and to develop from this condition certain structural properties of these continua and of hereditarily decomposable (indecomposable) continua. Henderson's result is arrived at as a direct consequence of the structural properties. In addition, examples are given to answer in the negative the following questions: *Question* 1. Does every snakelike continuum contain an arc or a psuedo arc? *Question* 2. Does every hereditarily decomposable continuum contain a cut point?

A *chain* is a finite collection of open sets $d_1, d_2, \cdots, d_l$ such that d_i intersects d_j if and only if $|i-j| \leq 1$. The condition is not imposed that the links be connected. If the links are of diameter less than ϵ, the chain is called an ϵ-chain. A compact continuum is called snakelike if for each positive ϵ it can be covered by an ϵ-chain.

1. A characterization of decomposable (and indecomposable) snakelike continua.

Definition. If M is a snakelike continuum, then the sequence of chains $\{C_i\}$ is said to be a defining sequence of chains for M if for each i: (1) C_i is a chain such that each link of C_i is a open set of diameter less than $1/i$; (2) there exists a δ such that the minimum distance between two non-adjacent links is greater than δ; (3) C_{i+1} is a closed refinement of C_i (i.e., the closure of each link of C_{i+1} lies in a link of C_i); (4) each link of C_i contains some point of M.

Lemma 1. *If M is a snakelike continuum, there exists a defining sequence of*

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