GENERALIZED CONDENSATION POINTS

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Blumberg [2], [3] introduced the concept of a set inexhaustibly approaching a point. This was generalized by Cargal [4] and Block and Cargal [1] to a point which is λ approached by a set. These ideas appear very closely connected with limit points of a set. In this paper we shall attempt to set these approach properties in a topological setting so that if x is "approached" by a set, it will be a limit point of this set in an appropriate topology.

A point x in the reals might be said to be non-denumerably approached by a set E if every neighborhood of x intersects E in a non-denumerable set. If this is the case, then x is a condensation point of E. A point x might be termed a point of non-null approach by E if every neighborhood of x intersects E in a set of positive outer Lebesgue measure. If this is the case, E is metrically dense at x, and x could be thought of as a generalized condensation point. These ideas are generalized and unified in the following way: With a given topological space we assume there is given a family of subsets satisfying certain conditions. Then a generalized condensation point of a set E will be a point such that each neighborhood of the point intersects E in a set contained in the family. We show that a topology can be defined in terms of the original topology and the family so that the generalized condensation points of a set are exactly the set of limit points of the set in the new topology. Some properties of this new topology are obtained and it is shown, Theorem 4, that the classes of continuous functions on both the new and old topological spaces to a regular space coincide. In case the original topological space is either the reals or complex numbers with the usual topology, it is shown that the classes of differentiable functions coincide.

1. **The derived set.** Frechét [6], Sierpinski [8], Riez [7] and Chittenden [5] among others, have studied the possibility of defining a topological space in terms of the derived set of a subset, i.e., the set of limit points. It is known that if X is a set and d a function defined on the class of all subsets of X satisfying

(a) $d(\phi) = \phi$
(b) if $E \subset F$, then $d(E) \subset d(F)$
(c) $d(E \cup F) \subset d(E) \cup d(F)$
(d) $d(d(E)) \subset d(E)$,

then the function d defines a closure operator on X, where the closure, E^c, of a set E is defined by $E^c = E \cup d(E)$. Then if $\mathfrak{J}$ is the topology defined by taking a set U to be open iff $(X - U)^c = X - U$, for any set $E \subset X$, $\bar{E} = E^c$

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