SOME INTERPOLATORY PROPERTIES OF TCHEBICHEFF POLYNOMIALS; (0, 1, 3) CASE.

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1. Recently P. Turán has studied the case of (0, 2) interpolation when the abscissas x_i

$$-1 \leq x_n < x_{n-1} < \cdots < x_1 \leq 1 \tag{1.1}$$

are the zero of

$$\pi_n(x) = (1 - x^2)P'_{n-1}(x) = -n(n-1)\int_{-1}^{x} P_{n-1}(t)\,dt \tag{1.2}$$

where $P_n(x)$ denotes the Legendre polynomial of degree n with the normalization

$$P_n(1) = 1. \tag{1.3}$$

In (0, 2) interpolation he seeks to find the polynomial $f(x)$ of degree $\leq 2n - 1$ whose values at the abscissas x_i given by (1.1) are prescribed. He has shown that for n even, these polynomials exist and are unique, but for n odd they are infinitely many. Their explicit forms have been obtained [1], and it has been shown [2] that these polynomials converge uniformly to the given function under certain conditions.

Later G. Freud proved the convergence theorem of Turán under different conditions. Saxena and Sharma [4] have extended the results to (0, 1, 3) interpolation and Saxena [3] has further extended them to (0, 1, 2, 4) case.

In all this we observe that the abscissas (1.1) are taken to be the zeros of $\pi_n(x)$ given by (1.2). In fact, the theorem of Turán for general ultraspherical polynomials about the existence of such interpolatory polynomials in the (0, 2) case does not take into consideration the case of Tchebicheff abscissas.

The object of this note is to extend the results of Turán [5], [1] Saxena and Sharma [4] of (0, 1, 3) interpolation to Tchebicheff abscissas. We limit ourselves in this to proving their existence and obtaining their explicit forms. The investigation into the convergence problem will form the subject of a later study.

2. As usual we denote throughout this paper by

$$T_n(x) = \cos n\theta \quad \text{where} \quad \cos\theta = x, \tag{2.1}$$

the Tchebicheff polynomials of the first kind. Let us consider the set of numbers

$$-1 < x_n < x_{n-1} < \cdots < x_2 < x_1 < +1 \tag{2.2}$$

by which we shall denote the zeros of $T_n(x)$. We shall prove the following

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