TRANSFORMATION OF AN ANALYTIC FUNCTION OF SEVERAL VARIABLES TO A CANONICAL FORM

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1. Let $F(z, w)$ be an analytic function of (z, w) for small $|z|$ and $|w|$ and suppose F is not identically zero. Then the Weierstrass preparation theorem states that, if $F(0, 0) = 0$, there are integers $k \geq 0$ and $m \geq 0$ such that

$$F(z, w) = z^k \Phi(z, w)[w^m + f_1(z)w^{m-1} + \cdots + f_n(z)] \tag{1.1}$$

where Φ is analytic for small $|z|$ and $|w|$ and $\Phi(0, 0) \neq 0$, and $f_i(z)$ are analytic for small z and $f_i(0) = 0$.

A related result, which was sketched in [1], will now be proved here.

THEOREM 1.1. *Let $F(z, w)$ be analytic for small $|z|$ and $|w|$. Let $F(z, w) - F(z, 0)$ not be identically zero. Then there exist integers $k \geq 0$ and $m \geq 1$ and an analytic function $G(z, s)$ of (z, s) such that setting*

$$w = s + s^2 G(z, s) \tag{1.2}$$

in $F(z, w)$ gives

$$F(z, w) - F(z, 0) = z^k \sum_1^m F_j(z) s^j \tag{1.3}$$

where $F_j(z)$ are analytic for small $|z|$,

$$F_j(0) = 0, \qquad 1 \leq j < m$$

and $F_m(0) \neq 0$. If $F_0(z) = F(z, 0)$, then (1.3) takes the form

$$F(z, w) = z^k \sum_1^m F_j(z) s^j + F_0(z). \tag{1.4}$$

If z^k, $k \geq 0$, is the highest power of z which is a factor of $F(z, w) - F(z, 0)$, then

$$f(z, w) = z^{-k}[F(z, w) - F(z, 0)]$$

is analytic for small $|z|$ and $|w|$, and there is a least integer $m \geq 1$ such that

$$\frac{\partial^m f}{\partial w^m}(0, 0) \neq 0.$$

Hence Theorem 1.1 is a consequence of the following theorem.

THEOREM 1.2. *Let $f(z, w)$ be analytic in (z, w) for small $|z|$ and $|w|$, let $m \geq 1$ and let*

$$\frac{\partial^j f}{\partial w^j}(0, 0) = 0, \qquad 0 \leq j < m, \tag{1.5}$$

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