RECURRENT PROPERTIES OF CONSERVATIVE MEASURABLE TRANSFORMATIONS

Dedicated to the memory of Mr. Wong Shou-Shiu, late mathematics and science teacher, Pui Ying Middle School, Canton, China

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Consider a fixed measure space (X, M, m), i.e., a set X together with a given sigma—algebra M of subsets of X and a measure m defined on M. The subsets of X belonging to M will be called measurable. In this note T is always a measurable transformation of X into itself, which means, by definition, that the inverse image $T^{-1}(A)$ of a set A is measurable whenever A is measurable. It is not assumed that T is either one-to-one or has a measurable inverse. We shall call T compressible if there exists a measurable set A such that $A \subset T^{-1}A$ and $m(T^{-1}A - A) > 0$; in the contrary case T is called incompressible. T is called recurrent if, for every measurable set E, almost every point of E returns to E under the action of T; strongly recurrent if, for every measurable set E, almost every point of E returns to E infinitely many times under the action of T.

In this note we shall establish the following:

THEOREM 1. *T is incompressible if and only if T^n is incompressible, n being any positive integer.*

The implication of this result is interesting. It is very easy to see from the definition of incompressibility and the Recurrence Theorem that T is incompressible if and only if T is recurrent [1; 10–11]. But if T is incompressible, and if $T^2, T^3, \cdots T^n \cdots$ are also incompressible, then T is strongly recurrent [1; 10–11]. Hence Theorem 1, together with the above remarks, implies the following theorem:

THEOREM 2. *Whenever T has one of the following properties:*

(i) *incompressibility,*
(ii) *recurrence,*
(iii) *strong recurrence,*
$T, T^2, T^3, \cdots T^n, \cdots$ *have all these properties.*

F. B. Wright [3], [4] has independently established that the incompressibility of T implies the strong recurrence of T without proving the incompressibility of the powers of T. His proof is elegant, however it yields no information about T^n.

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