BANACH ALGEBRAS OF MULTIPLIERS

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1. **Introduction.** Let A be a commutative semi-simple Banach algebra with maximal regular ideal space $\mathfrak{M}(A)$. In the Gelfand theory A is represented isomorphically by an algebra $\hat{A}$ of complex-valued functions on $\mathfrak{M}(A)$. S. Helgason in [5] considered Banach algebras A with the property that $\hat{A}$ is an ideal in $\mathcal{C}(\mathfrak{M}(A))$. Even when this is not the case, information about A is obtained by examining the largest sub-algebra of $\mathcal{C}(\mathfrak{M}(A))$ in which $\hat{A}$ is an ideal.

Let A^m be the set of complex-valued function f on $\mathfrak{M}(A)$ such that $f\hat{A} \subset \hat{A}$ (pointwise multiplication). Each f in A^m induces a unique bounded operator $\underline{f}$ on A. The set $\underline{A}^m$ of all such operators is a commutative semi-simple Banach algebra under the uniform operator norm. Further, let $\underline{A}_0^m$ designate the commutative semi-simple Banach algebra of operators induced on A by functions in $A^m \cap \mathcal{C}_0(\mathfrak{M}(A))$. $\underline{A}^m(\underline{A}_0^m)$ is called the algebra of multipliers (multipliers vanishing at infinity) of A.

In particular, if A is $L_1(G)$, where G is a locally compact Abelian group, then $\underline{A}^m$ is isometrically isomorphic to $M(G)$, the Banach algebra of all complex-valued regular Borel measures on G. Consequently, the algebra of multipliers is also of intrinsic interest as a natural generalization of the algebra of measures. Quite a few analogues of theorems about $M(G)$ remain true for $\underline{A}^m$ even in the absence of the underlying group G.

In this paper our purpose is to relate properties of A, A_0^m and A^m. We establish in §2 the basic theory, in §3 we give various characterizations of multipliers as operators on A and apply these to study possible new multiplications on A which leave the maximal regular ideals of A unchanged, in §4 we indicate some more or less immediate consequences of certain specializations of A or A^m, in §5 we use a result of Arens and Singer [1] to obtain values of multipliers as boundary integrals and by means of this representation to compare maximal regular ideal spaces: $\mathfrak{M}(A)$, $\mathfrak{M}(A_0^m)$ and $\mathfrak{M}(A^m)$.

In papers to follow we plan to discuss similarities between A^m and $M(G)$: isomorphism theorems, isometric multipliers and an analogue of Eberlein's characterization of Fourier-Stieltjes transforms of measures [4].

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2. **Basic properties.** The fact that the algebra of multipliers is a commutative semi-simple Banach algebra is probably well known. It was indicated by Helgason in [5] and was developed by Wang in [8]. However, for the sake of

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