ACCUMULABILITY AND INFINITE MATRICES

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We define a concept generalizing the concept of summbability. Let $A = (a_{in})$, $i = 1, 2, \cdots ; n = 1, 2, \cdots$, be a matrix of complex numbers. The sequence $\{y_n\}$ will be called A-accumulable if infinitely many terms of the sequence of auxiliary means $t_i = \sum_{n=1}^{\infty} a_{in}y_n$ are defined, and the defined sequence has exactly one limit point in the finite part of the plane. The first theorem generalizes [8] and is related to results in [1].

THEOREM 1. *Let $A = (a_{in})$ be an infinite matrix of complex numbers such that A accumulates every convergent sequence. Let A have bounded columns. Then the rows, possibly infinite in number, for which $\sum_{n=1}^{\infty} | a_{in} |$ diverges can be stricken from A to leave an infinite matrix which actually sums every convergent sequence. Even if the columns of A are not bounded, there exists an infinite number of rows such that the matrix consisting of only those rows sums every convergent sequence.*

Proof. We first prove that an infinite number of $\sum_{n=1}^{\infty} | a_{in} |$ converge, and then that these sums are bounded. If all but a finite number of $\sum_{n=1}^{\infty} | a_{in} |$ diverge, we can construct a sequence $\{b_n\}$ convergent to zero and such that all but a finite number of $\sum_{n=1}^{\infty} a_{in}b_n$ diverge. The construction is straightforward and omitted. Let i_j, $j = 1, 2, \cdots$, be the set of indices of those rows of A for which $\sum_{n=1}^{\infty} | a_{in} |$ converges.

Let $K_j = \sum_{n=1}^{\infty} | a_{i_j,n} |$; we may suppose K_j goes to infinity with j. We will construct a convergent sequence $\{c_n\}$ whose sequence of i_j-th auxiliary means has more than one limit point in the finite plane. We may assume that the i_j-th row is finite by replacing all terms in the i_j-th row by zero as soon as the sum of the first N_j absolute values of the entries differs from K_j by, say, $1/j$ or less. For this change alters the set of limit points of the sequence of i_j-th auxiliary means of no convergent sequence. We first assume $\{a_{i_j,n}\}$ as a sequence in j for fixed n is bounded. We will choose a sequence $\{c_n\}$ convergent to zero and such that an infinite number of auxiliary means are $+ 1$, and an infinite number are $- 1$. Let $L_1 = K_{j_1} \geq 1$ and choose $c_n =$

$$| a_{i_{j_1},n} | (a_{i_{j_1},n}L_1)^{-1}, \qquad 1 \leq n \leq N_{j_1}, \quad \text{with} \quad | z | z^{-1} = 1 \quad \text{if} \quad z = 0.$$

Choose $j_2 > j_1$ such that $L_2 \geq 2$ where

$$L_2 = \left(\sum_{n=N_{j_1}+1}^{N_{j_2}} | a_{i_{j_2},n} | \right) \left(1 + \sum_{n=1}^{N_{j_1}} a_{i_{j_2},n}c_n \right)^{-1}.$$

L_2 may be infinite. Define

$$c_n = -| a_{i_{j_2},n} | (a_{i_{j_2},n}L_2)^{-1}, \qquad N_{j_1} + 1 \leq n \leq N_{j_2}.$$

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