LIGHT OPEN MAPS ON n-MANIFOLDS

By Philip T. Church and Erik Hemmingsen

Introduction. This paper describes an investigation of light and/or interior (i.e. open) maps on n-manifolds. Theorems are proved which restrict the local behavior of some of these maps, and certain kinds of sets are ruled out from the class of possible branch sets (see Definition 3).

Light interior maps on 2-manifolds have been studied extensively by Whyburn [9] and [10] and by Stoilow [5], and have been shown to be topologically equivalent to analytic maps [5; 121]. The n-dimensional maps, for arbitrary n, are of particular interest because (1) some questions about them appear related to problems about groups acting on n-manifolds; and (2) the Monotone-Light Factorization Theorem of Eilenberg [9] suggests that a knowledge of such maps will help in the study of arbitrary interior maps of n-manifolds into n-manifolds. Indeed, in the latter case, if M and N are compact n-manifolds, if f is an interior map of M onto N, and if $g: M \to X$ and $h: X \to N$ are the factoring maps, we do not know whether X need be an n-manifold, but it will be compact metric and h will be light interior. Several of the results in this paper are true for such maps h; but, to keep the statements of theorems reasonably simple, the authors have avoided the most general forms.

A light interior map on a 2-manifold behaves locally just as, for some $n = 1, 2, \cdots$, the analytic map $z \to z^n$ behaves at the origin. Let $g_n : E^3 \to E^3$ be the map sending each horizontal plane onto itself in the manner of $z \to z^n$; if we compactify and call the corresponding map h_n, $h_n : S^3 \to S^3$, we obtain "winding" about a 1-sphere (and about a 0-sphere on each compactified plane). If f is a simplicial light interior map whose domain and range are compact simplicial 3-manifolds, the work of Tucker [8] implies that its branch set consists of a finite number of 1-simplices around whose interiors the mapping behaves locally like h_n, for some n. (One might be tempted to conjecture that, as in the 2-dimensional case, every light interior map f on a 3-manifold M is locally the composition of maps, each of which is either a homeomorphism or a restriction of some h_n. The following example shows that this conjecture is false: let $g: S^2 \to S^2$ be a map with three branch points, one of local degree 3 and two each of local degree 2; let M and $f(M)$ be the suspensions (double cones) on these two 2-spheres, and let f be the natural extension of g to M.)

Each space discussed in this paper is separable metric. In addition f *will always denote a map of an n-manifold M into an n-manifold N.*

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