A MILDLY WILD IMBEDDING OF AN n-FRAME

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1. Introduction. An *n-frame* D in Euclidean three-dimensional space R consists of n simple arcs $a_i = op_i$, $i \equiv 1, \cdots, n \pmod n$ which are pairwise disjoint except for their common end point o. In accordance with the definition given in [1] an n-frame D is said to be *tamely imbedded in* R if there is a homeomorphism of R upon itself that transforms D into a polygonal n-frame, say into the *standard n-frame* defined in cylindrical coordinates by $\rho \leq 1$, $n\theta/2\pi$ integral, $z = 0$; if there is no such homeomorphism, D is said to be *wildly imbedded.*

If we delete from D all of the i-th arc a_i except the end point o we get an $(n - 1)$-frame that we will denote by D_i. It is not difficult to construct a wildly imbedded n-frame D such that each of the derived $(n - 1)$-frames $D_1, \cdots, D_n$ is also wildly imbedded. The object of this paper is to give an example of a wildly imbedded n-frame D each of whose derived $(n - 1)$-frames $D_1, \cdots, D_n$ is tamely imbedded. The wildness of the imbedding of such an n-frame is therefore rather mild. A mildly wild 2-frame was given in [1], and a mildly wild 3-frame was attempted in [2]. However the proof in [2] that the triod (= 3-frame) T' is wildly imbedded is incorrect because it is based on an incorrect presentation of the group G_p. A corrected presentation of G_p has the relations (A), (B) and (C) as given in [2; 265] and relations

$$(\mathrm{V}_1) \quad c_k\, b_k\, a_k = 1 \quad \text{for} \quad k \quad \text{even}$$

$$(\mathrm{V}_2) \quad a_k\, b_k\, c_k = 1 \quad \text{for} \quad k \quad \text{odd}$$

$$(\mathrm{V}_3) \quad c_{k+1}\, a_{k+1}\, b_k = 1 \quad \text{for} \quad k \quad \text{odd.}$$

Unfortunately with the corrected presentation it is easy to show that both $\pi(F_n' - T')$ and its image in $\pi(F_1' - T')$ are free of rank 2, so wildness of the imbedding of T' cannot be concluded by this method. In fact an autohomeomorphism of R that carries Doyle's triod [2] onto a standard triod has been constructed by R. H. Bing.

Although the method of proof of wildness used in [2] does not work for the example in [2], it is a valid method and is the basis of our proof of the wildness of our example. Our example of a mildly wild n-frame is closely related to and was suggested by examples, of the type considered by H. Brunn [3], of n interlocked rings that fall completely apart when any one of them is dissolved.

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