# COHOMOLOGY GROUPS OF MAXIMAL ORDERS OF 𝔭-ADIC SIMPLE ALGEBRAS

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**1.** The cohomology groups of principal orders of 𝔭-adic number fields have period 2 [5]. This is also true for maximal orders of central simple algebras over 𝔭-adic fields.

Let $k$ be a 𝔭-adic number field, let $R$ be the ring of all 𝔭-integers in $k$, and let $\mathfrak{A}$ be a central simple algebra over $k$. A maximal order $\Lambda$ in $\mathfrak{A}$ over $R$ may be considered as an algebra over $R$. Therefore, for any two sided $\Lambda$-module $M$, we may consider cohomology groups $H^r(\Lambda, M)$ and homology groups $H_r(\Lambda, M)$ of $\Lambda$ in $M$ [1]. Then we have

$$H^{r+2}(\Lambda, M) \cong H^r(\Lambda, M) \qquad H_{r+2}(\Lambda, M) \cong H_r(\Lambda, M)$$

for $r \geq 1$. We shall prove it in the following sections. Mr. H. Ogawa has also proved it for cohomology groups with coefficients in $\Lambda/\pi^s\Lambda$ by using relative cohomology theory [6].

**2.** We shall begin with division algebras. Let $k$ and $R$ be as above and let $\mathfrak{K}$ be a central division algebra over $k$ of degree $n^2$. Then there exists a unique maximal order $\Lambda$ of $\mathfrak{K}$ over $R$. Furthermore there exist a prime element $\pi$ and a root $\zeta$ of unity such that

$$\Lambda = \sum R\pi^i\zeta^j \quad (\textit{direct}), \qquad i, j, = 0, 1, 2, \cdots, n-1, \tag{1}$$

$$\pi^n = p, \qquad \pi\zeta\pi^{-1} = \zeta^{q^r}, \qquad (r, n) = 1,$$

where $p$ is a prime element in $k$ and $q = \#(R/(p))$ [2]. In other words, $k(\zeta)/k$ is unramified cyclic, and the automorphism

$$\rho : \zeta \longrightarrow \pi^{-1}\zeta\pi \tag{2}$$

is a generating automorphism of $k(\zeta)$ over $k$. We denote the Galois group of $k(\zeta)/k$ by $G$.

Now, we consider the structure of $R[\zeta] \otimes_R R[\zeta]$. Any $\sigma \times \tau \in G \times G$ induces an automorphism $a(\zeta) \otimes b(\zeta) \rightarrow a(\zeta)^\sigma \otimes b(\zeta)^\tau$ of $R[\zeta] \otimes_R R[\zeta]$. We shall denote images of this automorphism by $(a(\zeta) \otimes b(\zeta))^{\sigma\times\tau}$.

Proposition 1. *The unit element* $1 \otimes 1$ *of* $R[\zeta] \otimes_R R[\zeta]$ *has a decomposition*

$$1 \otimes 1 = e_1 + e_\sigma + \cdots + e_\tau, \qquad 1, \sigma, \cdots, \tau \in G, \tag{3}$$

*into orthogonal minimal idempotents* $e_1, e_\sigma, \cdots, e_\tau$ *which satisfy*

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