RETRACTION PROPERTIES OF THE ORBIT SPACE OF A COMPACT TOPOLOGICAL TRANSFORMATION GROUP

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1. Introduction. This paper is motivated by Floyd's result to the effect that whenever a finite group operates as a group of topological transformations on a compact finite dimensional absolute neighborhood retract (ANR), then the orbit space is also an ANR [9]. The question was later raised about the possibility of extending this theorem to the action of compact Lie groups [10]. Some extensions of this type have already been made [5], [6]. In this paper we do not solve the problem completely, but we reduce the conjectured generalization of Floyd's theorem to what at first appears to be an unrelated conjecture about compact connected simple Lie groups.

Conjecture I. *If* (G, X) *denotes a compact Lie group acting on a compact connected finite dimensional* ANR, *and if* (G, X) *has a finite number of conjugacy classes of isotropy subgroups, then the orbit space* X/G *is also an* ANR. *Furthernore, if* X *is an absolute retract* (AR), *then so is the orbit space* X/G.

We shall now state a conjecture, the validity of which implies Conjecture I.

Conjecture II. *For every compact connected simple Lie group* G *there is a transformation group* (G, M) *with a finite number of conjugacy classes of isotropy groups on a compact space* M *such that* (G, M) *has no stationary points and such that* $H^t(M; Z) \approx H^t(M/G; Z) = 0$ *for* $0 \leq t < \infty$.

The fact that II implies I is not immediately obvious, and this paper is devoted to proving this point. Fortunately, Floyd has verified the second conjecture for $SO(2n + 1)$, $n \geq 1$. We shall not discuss these examples which will appear elsewhere [12]. It will follow that the first conjecture is valid if the identity component of G is Abelian or if it is isomorphic to SO(3), SO(4) or to SO(5).

Some progress is made on a related question raised by Montgomery, who asked if the orbit space of a compact Lie group acting on Euclidean space is always contractible. Again, the answer is affirmative if Conjecture II is valid, therefore the problem is settled if the identity component of the group is Abelian or if it is SO(3), SO(4) or SO(5).

In addition to the results mentioned above we obtain specific information about the local Betti groups (rational cohomology) of the orbit space. No restriction is made on the group, other than it be a compact Lie group, for theorems about the local Betti of the orbit space.

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