A CHARACTERIZATION IN E^3 OF THE EVERYWHERE REGULAR SOLUTION OF THE REDUCED WAVE EQUATION

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1. **Introduction.** In 1958 the author established [5] a theorem which asserts that there exists on E^2, the Cartesian (x, y)-space, a unique solution of the 2-dimensional reduced wave equation for which the integral of the solution along the semi-infinite rays of a pencil of lines can be prescribed. In fact the following result was established.

Theorem. *There exists an everywhere defined unique C^2 solution on E^2 of the reduced wave equation*

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u = 0 \qquad (x^2 + y^2 < \infty) \tag{1.1}$$

for which uniformly in θ the

$$\int_0^\infty u(x, y)\, dr = F(\theta) \qquad (x + iy = re^{i\theta}) \tag{1.2}$$

where the arbitrarily prescribed F is of period 2π and satisfies a uniform Hölder condition with exponent $\alpha > \frac{1}{2}$. Furthermore, the solution has the integral representation

$$2\pi u(x, y) = \int_0^{2\pi} F(\phi) \cos (r \sin (\phi - \theta))\, d\phi + \pi^{-1} \int_0^{2\pi} d\phi F(\phi) \int_0^{\pi/2} \cdot \frac{[\sin (r \sin (\tau + \phi - \theta)) + \sin (r \sin (\tau - \phi + \theta))]\, d\tau}{\sin \tau}. \tag{1.3}$$

In this paper it is our intent, with respect to the 3-dimensional reduced wave equation, to prove the following result.

Theorem. *There exists an everywhere defined unique C^2 solution on E^3 of the reduced wave equation*

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + u = 0 \qquad (x^2 + y^2 + z^2 < \infty) \tag{1.4}$$

for which uniformly in (θ, ϕ) the

$$\int_0^\infty r^{\frac{1}{2}} u(x, y, z)\, dr = F(\theta, \phi) \qquad (x + iy = r \sin \theta e^{i\phi}, \quad z = r \cos \theta) \tag{1.5}$$

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